

*“Navigare necesse est,  
vivere non est necesse.”*

Pompeius

# **Navigation**

# What does navigation means?

„Navigation is the process of determining and maintaining a course or trajectory from one place to another. Processes for estimating one's position with respect to the known world are fundamental to it. The known world is composed of the surfaces whose locations relative to one another are represented on a map.”

C. R. Gallistel: *The Organization of Learning*. MIT Press/Bradford Books, MA, 1990

Thus, we need to know:

Where we are?

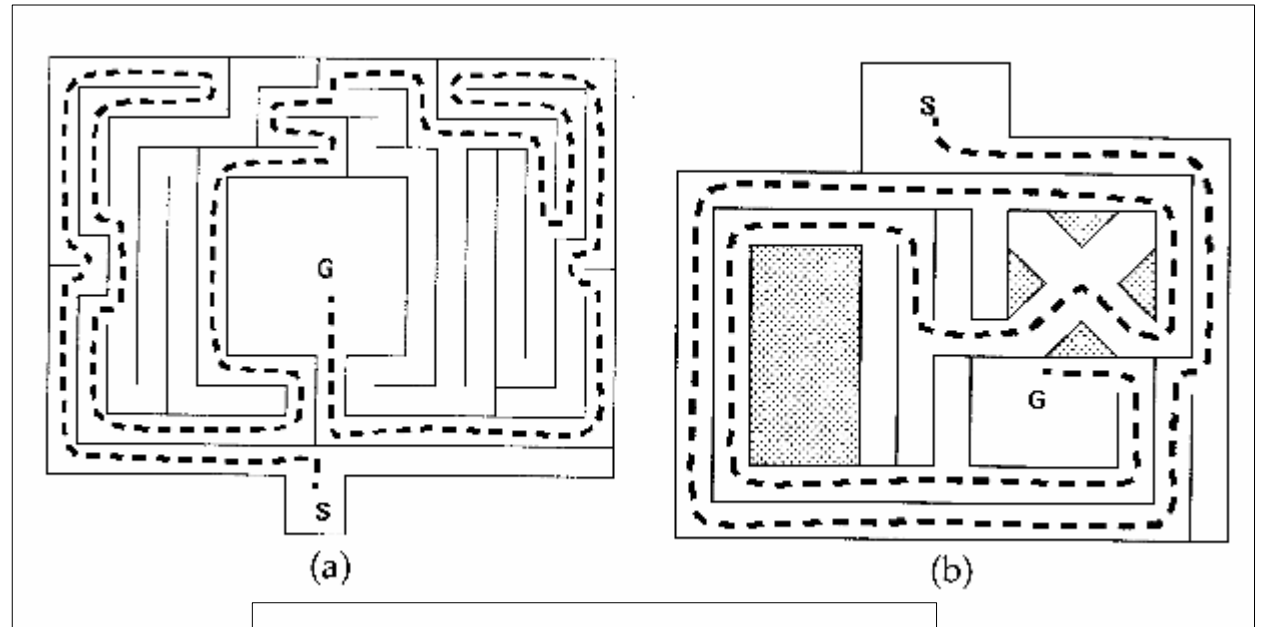
Where are the important places relative to me?

How to get there?

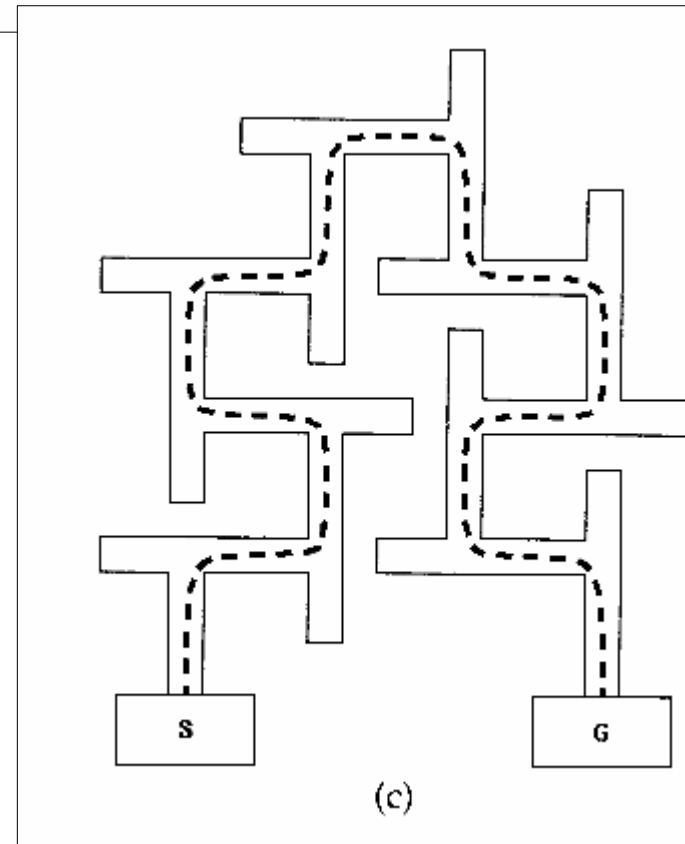
# Hierarchy of Navigation Strategies

	Information to be stored	External information required	Procedure
0. Random Navigation	none	none	Random wandering
1. Praxic Navigation	Position in a sequence	none	Execution of a pre-defined / learned sequence of actions
2. Dead reckoning	Actual position	none	Path integration
3. Target approaching / Guidance / Avoidance	Sensible property of / landmark configuration at the aim	Direct / Indirect sensation of the aim	taxis
4. Place triggered	1. Landmark configurations defining places 2. Local directional reference frames 3. Direction of the movement, that leads to the aim	Actual landmarks	Self-localization by place recognition Association of recognized places to the actions that leads towards the aim
5. Topological navigation	Set of landmark configurations linked by topological relationships	Actual landmarks	Graph searching, way finding
6. Metric Navigation	Set of landmark configurations linked by metrical relationships	Actual landmarks	Vector subtraction, trigonometrics

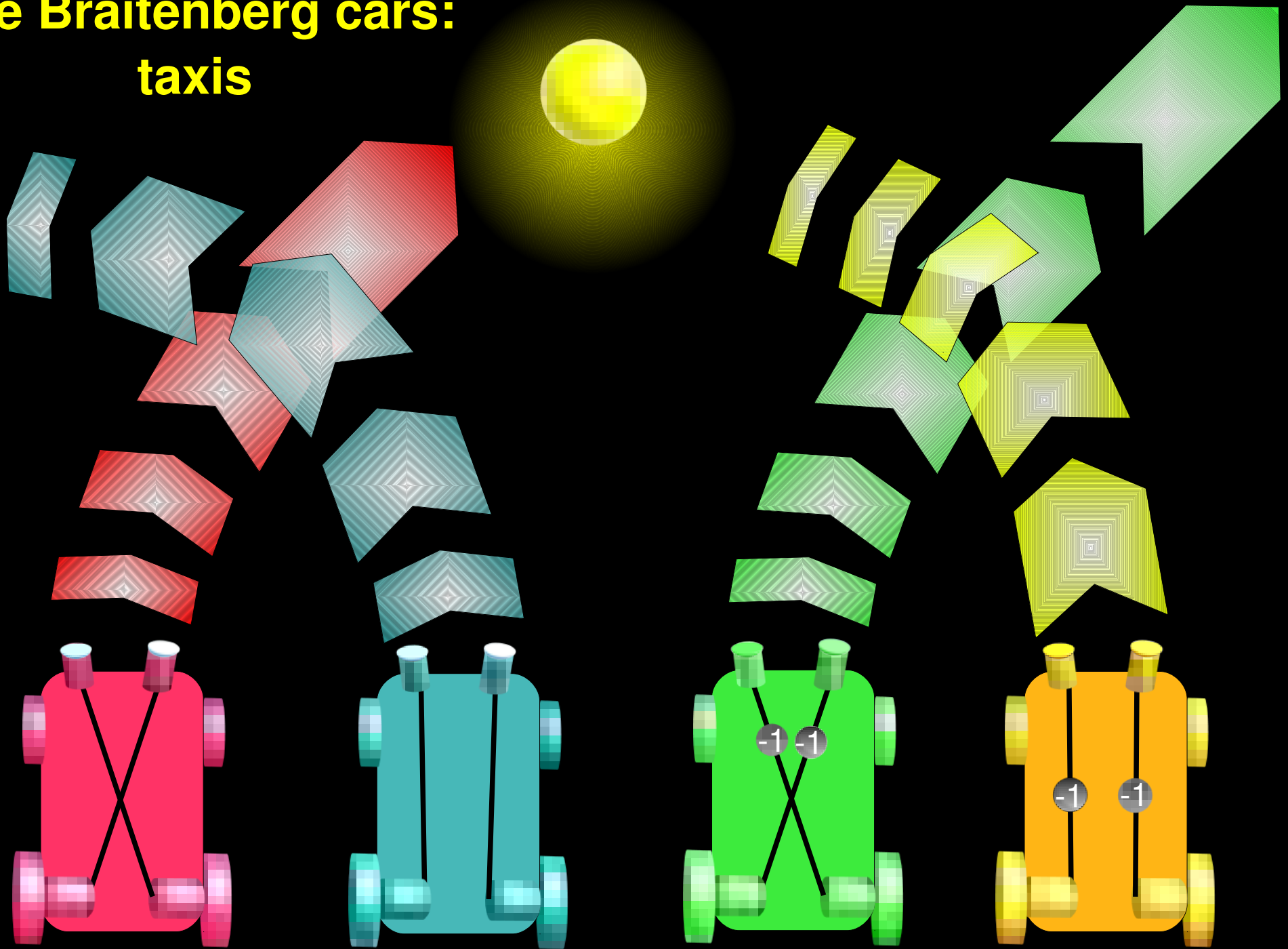
# Typical praxic tasks and solutions



No external clue or  
sensory input is required



# The Braitenberg cars: taxi



# Place recognition-triggered versus topological navigation

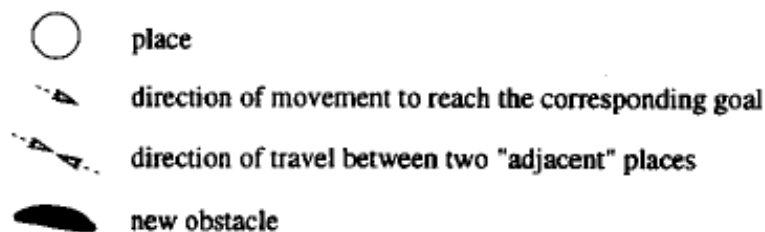
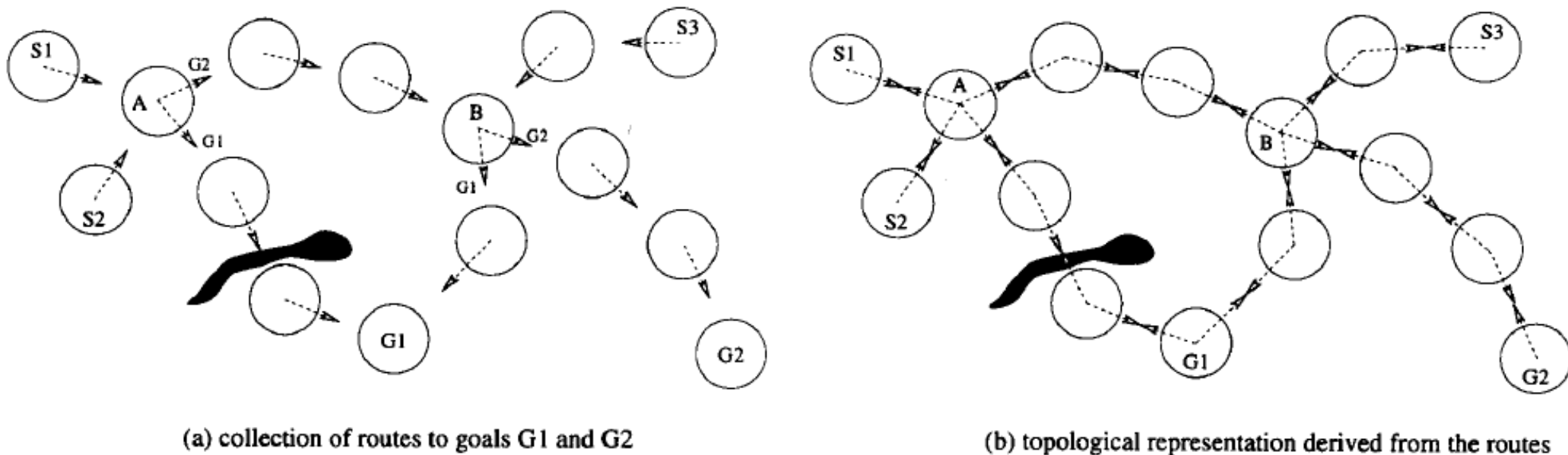
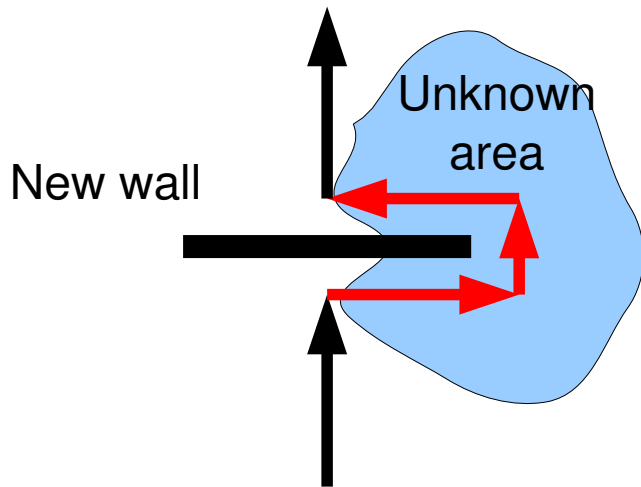


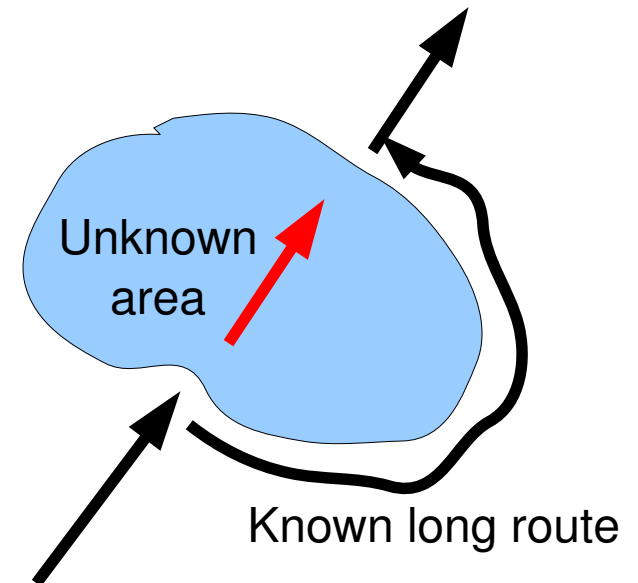
Fig. 3. (a) With the place recognition-triggered response strategy there can be an ensemble of intersecting routes. The animat is able to go from S1 to G1, from S2 to G2, and from S3 to G1. However, if there is a new obstacle on the way from S1 to G1, as on this figure, the animat is lost, because the route from S1 to G1 is unique (see also Fig. 2). (b) In contrast, if the animat merges its representations of routes into a topological representation, the animat can go back to place A, take the sub-route between places A and B, and take the sub-route from place B to the goal G1. The resulting path is the concatenation of three sub-sequences, derived from three different routes.

# Metric navigation

## Detour finding

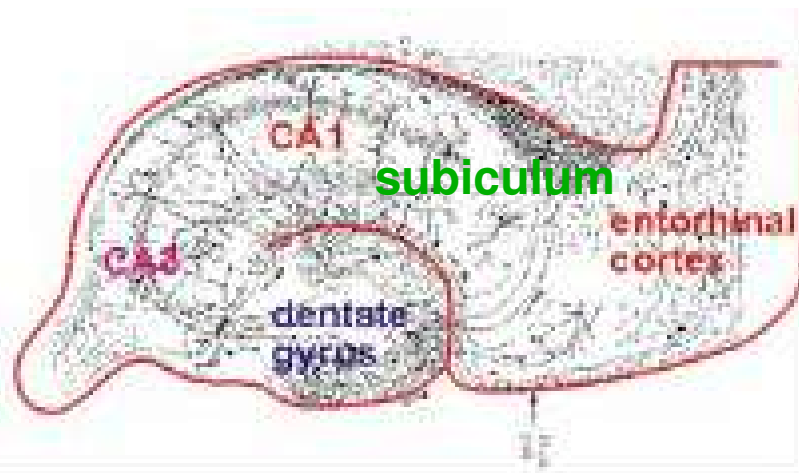
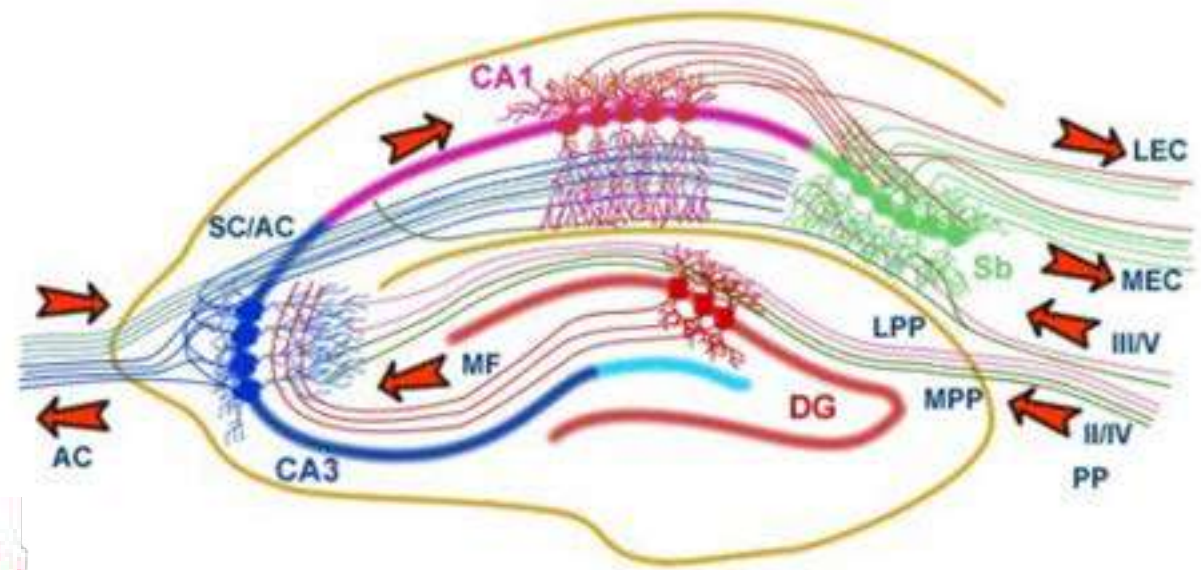
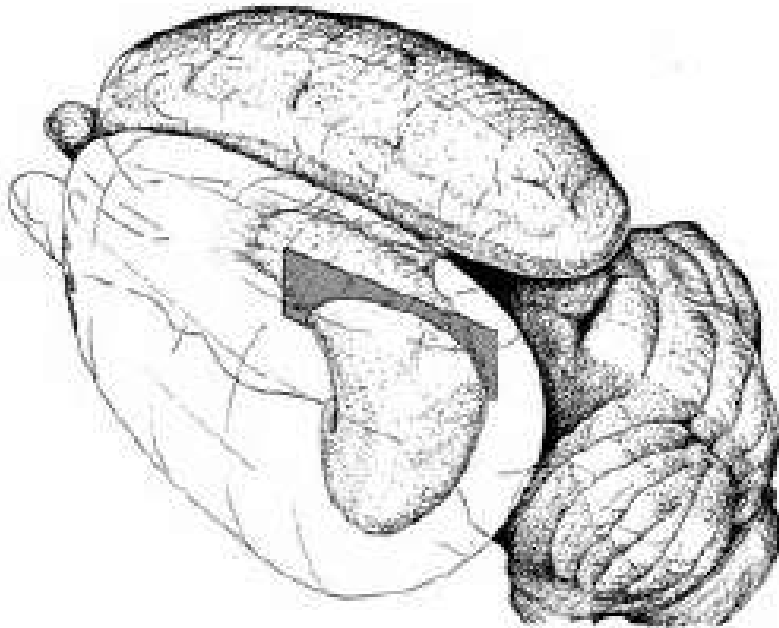


## Shortcut finding



# Hippocampus

- Place recognition is required to apply higher order navigation strategies
- Place cells in the hippocampus (Pyramidal cells in CA1 and CA3 region, 1971)



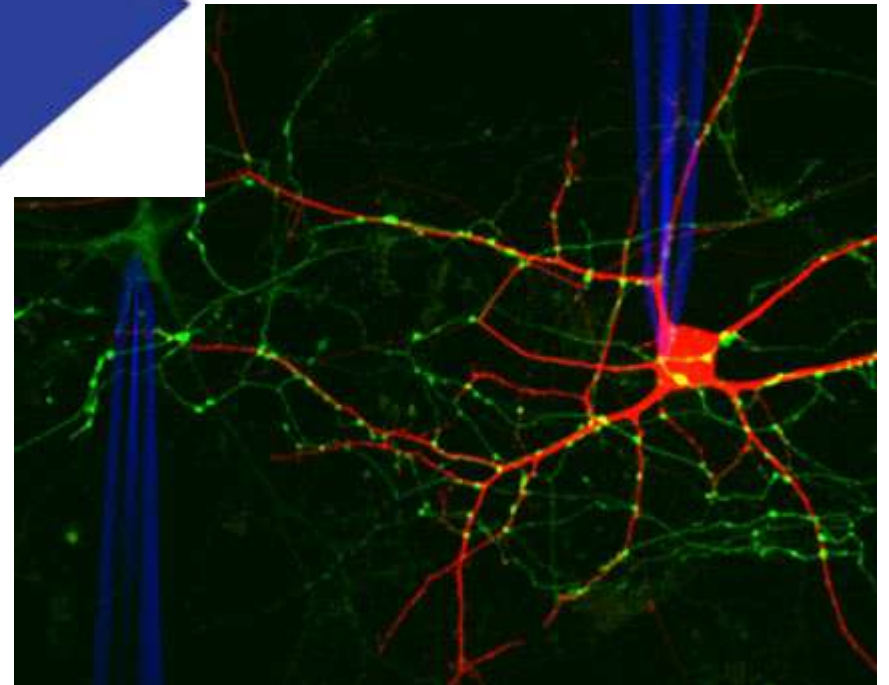
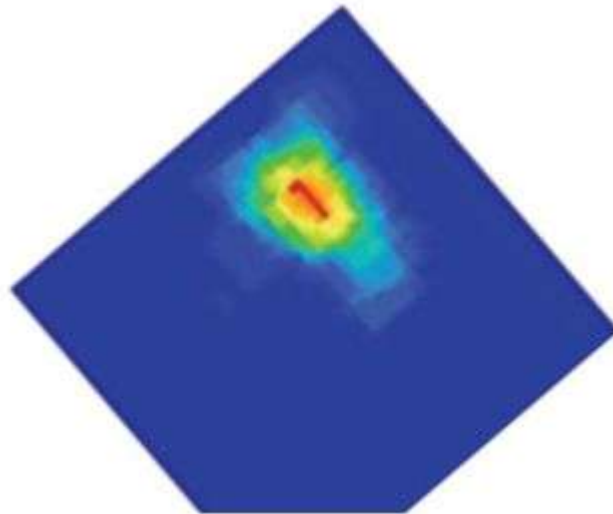
The Hippocampal Network: The hippocampus forms a principally uni-directional network, with input from the Entorhinal Cortex (EC) that forms connections with the Dentate Gyrus (DG) and CA3 pyramidal neurons via the Perforant Path (PP - split into lateral and medial). CA3 neurons also receive input from the DG via the mossy fibres (MF). They send axons to CA1 pyramidal cells via the Schaffer Collateral Pathway (SC), as well as to CA1 cells in the contralateral hippocampus via the Associational Commissural pathway (AC). CA1 neurons also receive input directly from the Perforant Path and send axons to the Subiculum (Sb). These neuron in turn send the main hippocampal output back to the EC, forming a loop.



# Hippocampus

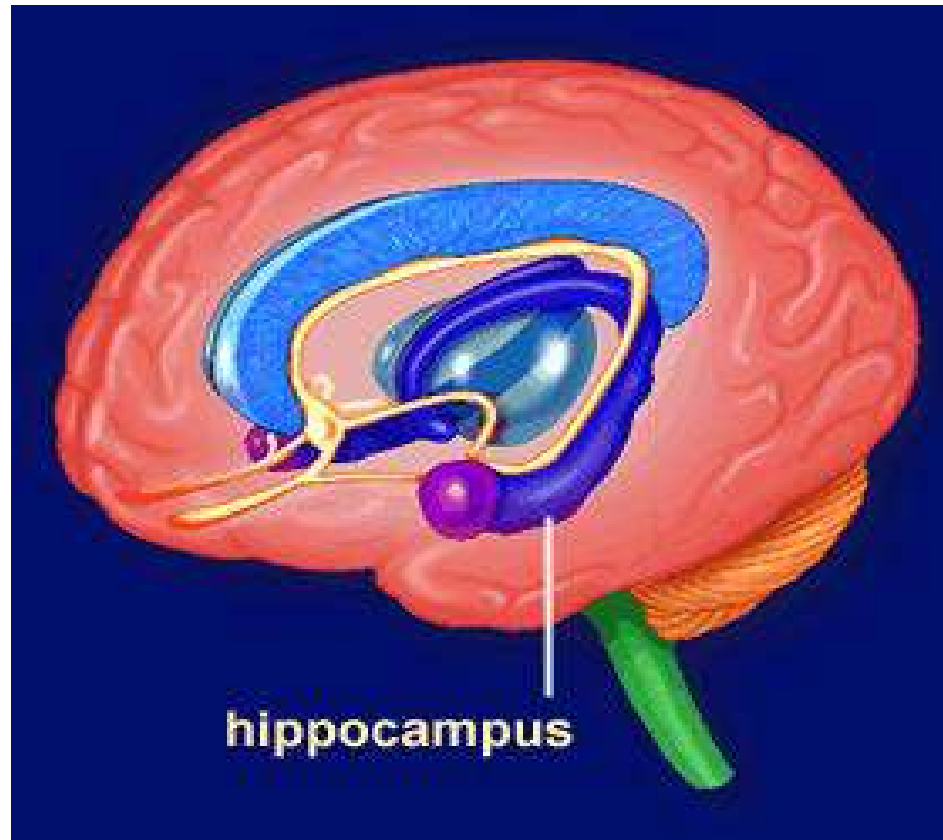
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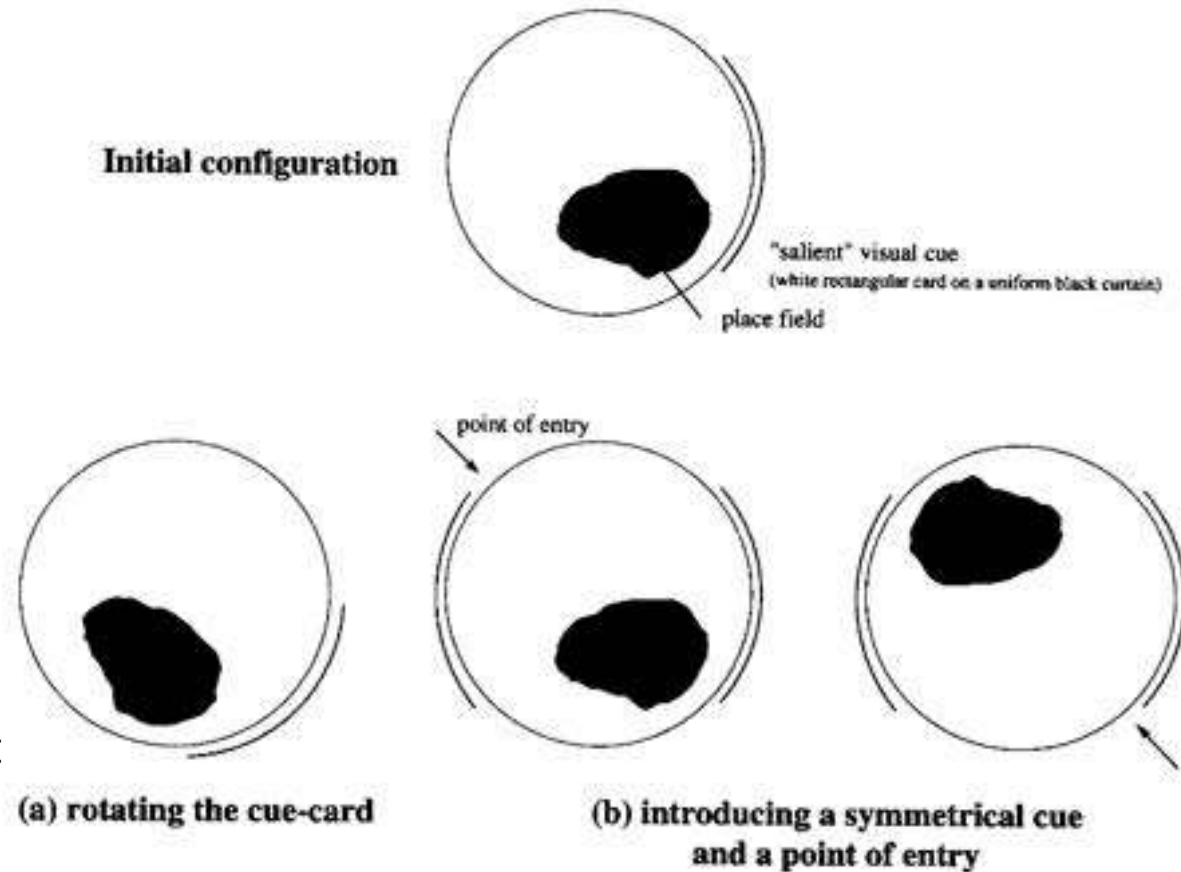
# Hippocampus in the human brain: episodic memory

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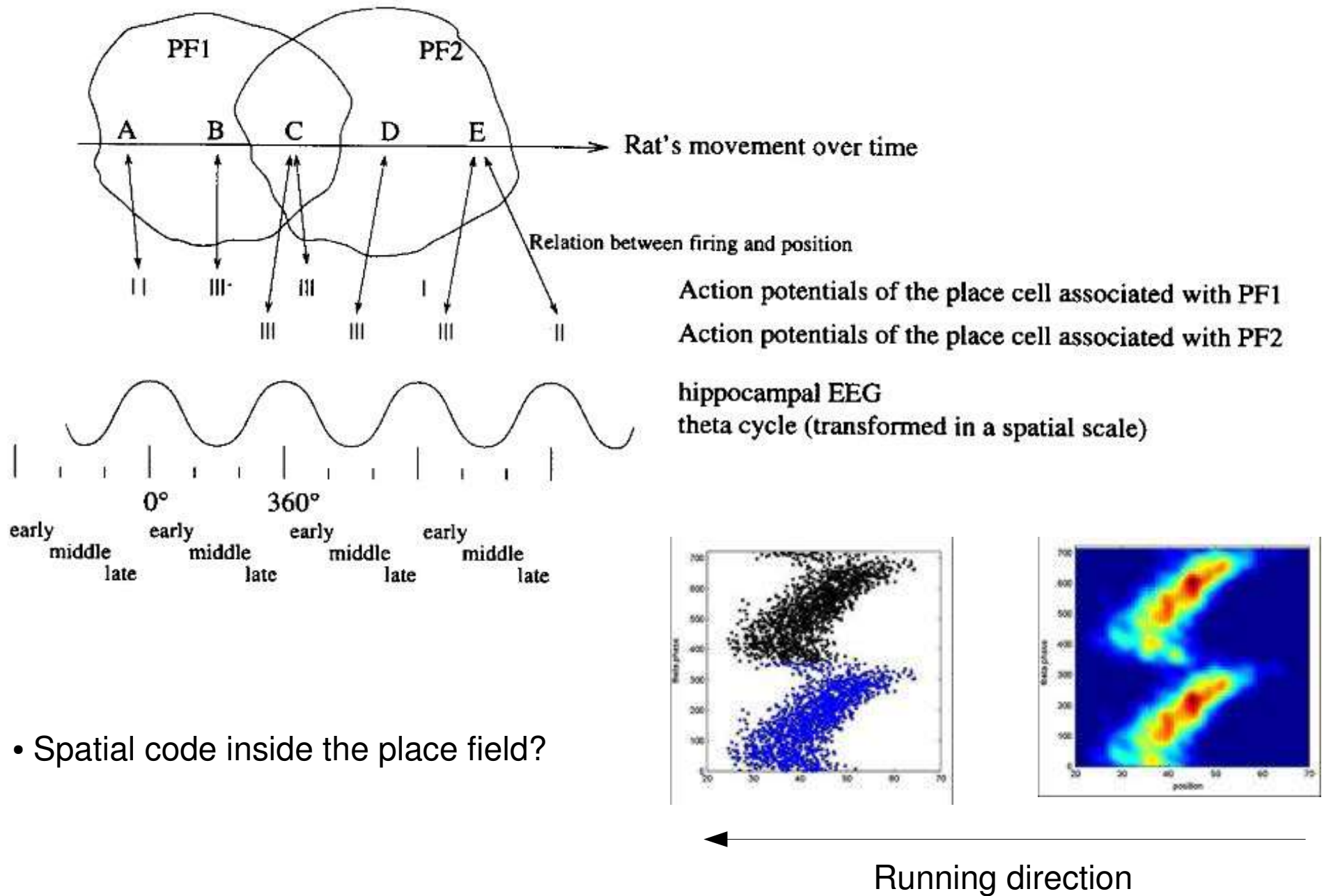


# What determines the position of a place field?

- Visual information
- but: in blind and deaf animals
- Tactile
- Olfactory
- Vestibular
- Memory traces
- Context
- Firing frequency is independent of the direction
- Independent of the aim
- Frequency coding

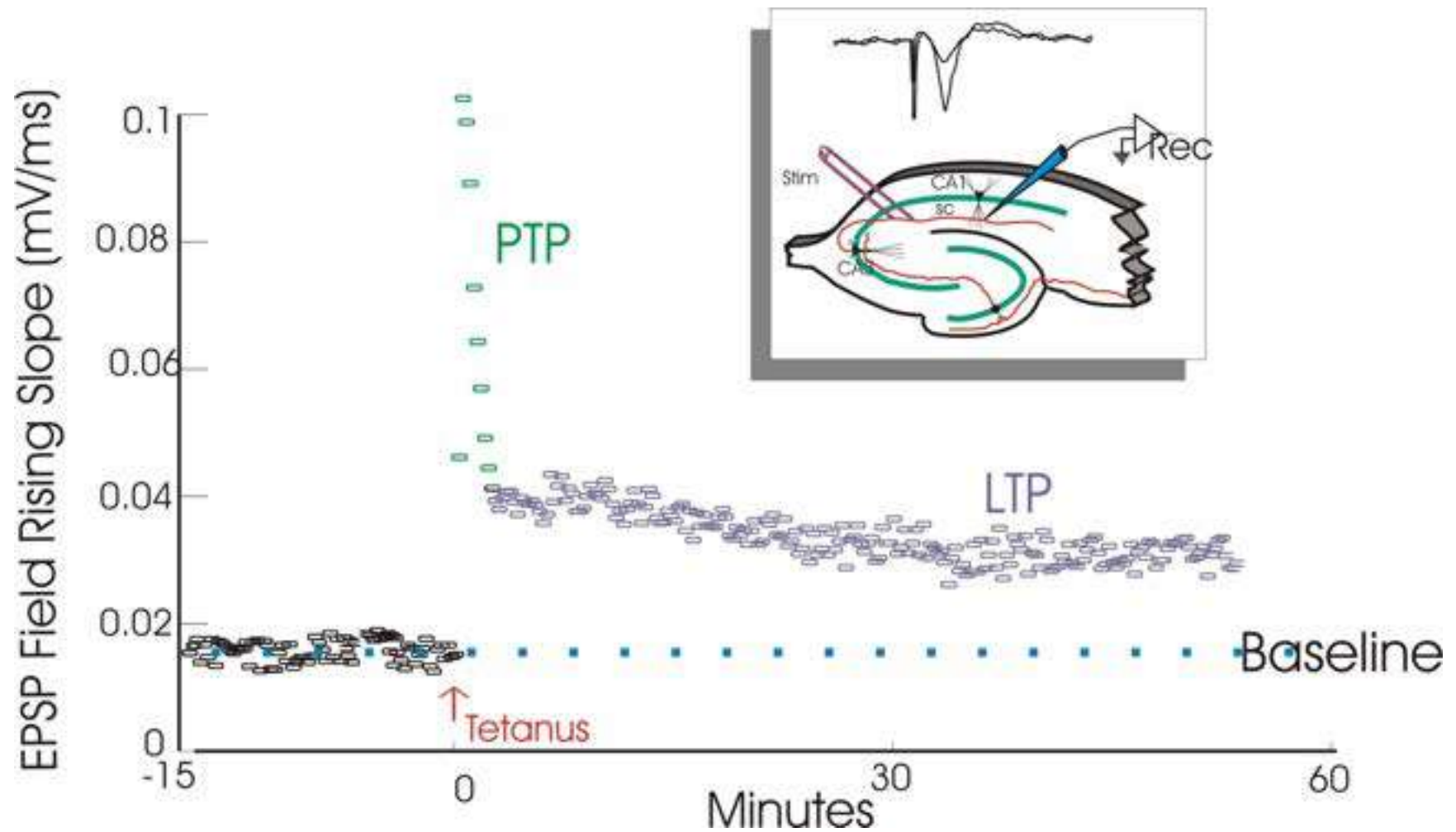


# Phase precession – phase code



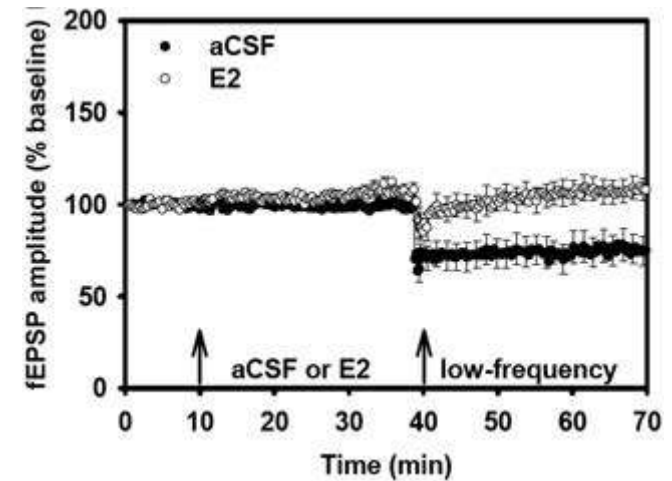
# Long Term Potentiation

- LTP – long term potentiation (1966): was discovered in the hippocampus.

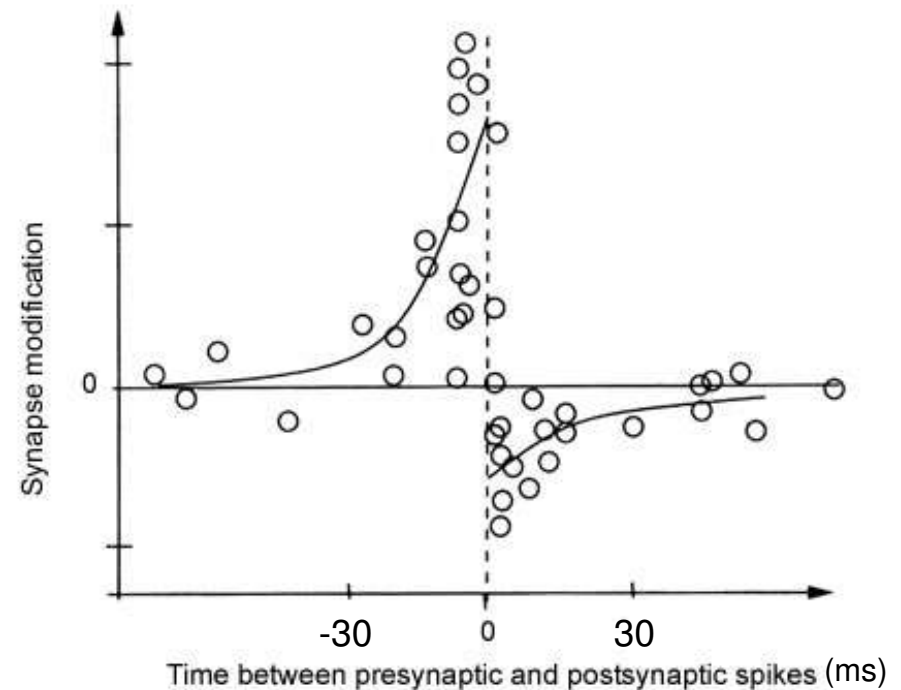


# Learning at cellular level

- LTD – long term depression: can be elicited
- by weak stimulus

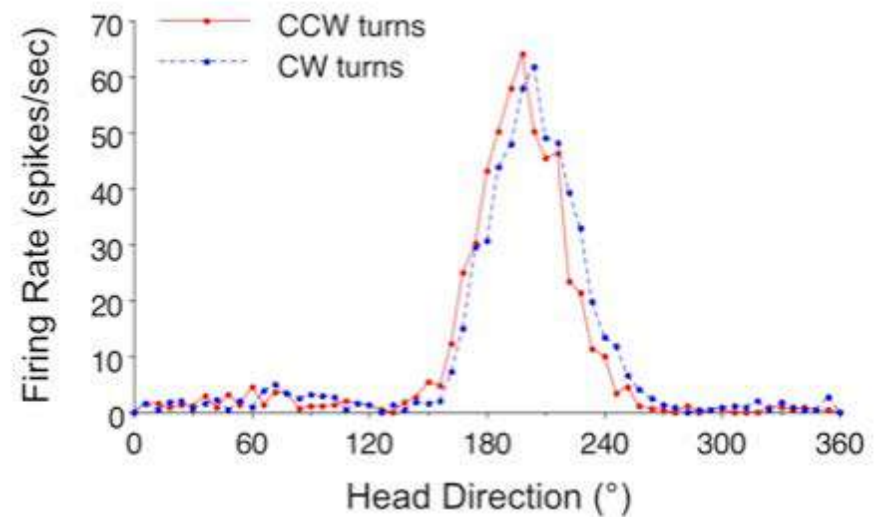
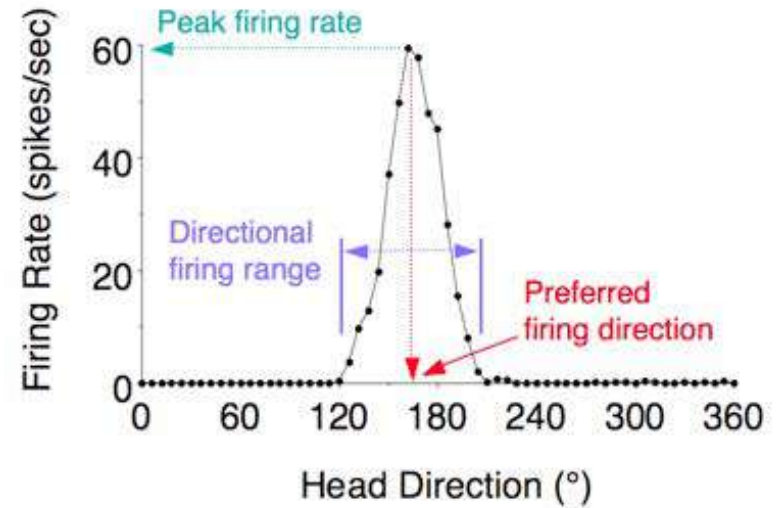


- STDP – spike timing dependent plasticity:



# Head direction cells

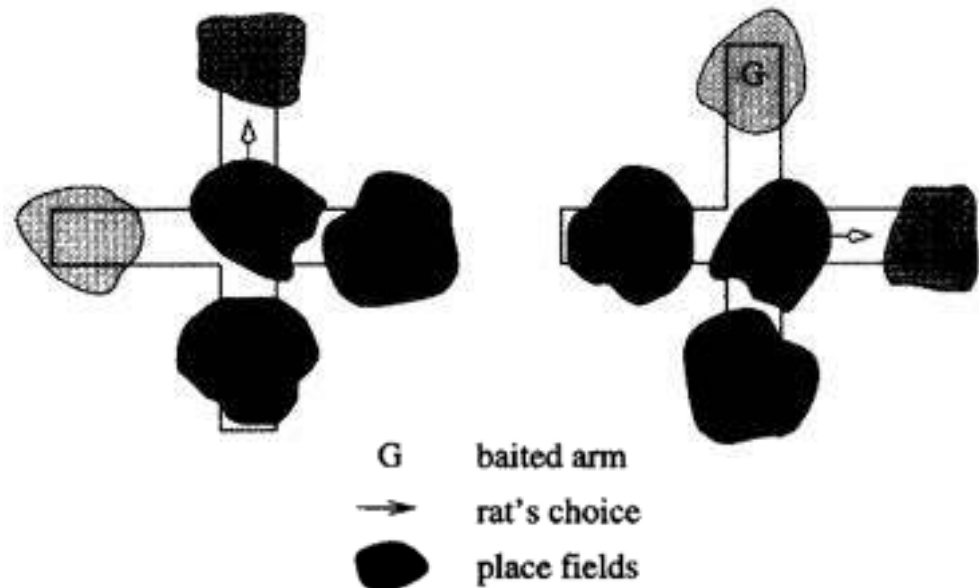
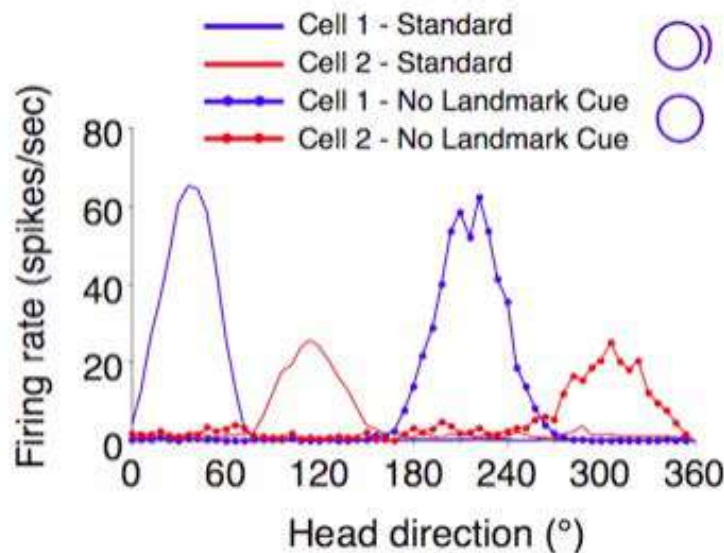
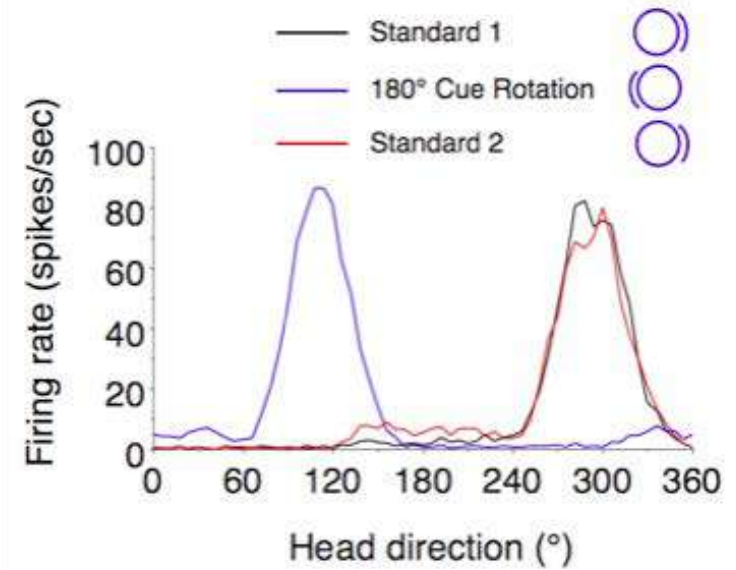
- Their firing correlate with the head direction
  - Independently of the position
  - During rest and movement
  - presubiculum, postsubiculum, posterior cortex, thalamus, striatum
- 
- In the thalamus: The future head direction is coded (~25 ms latter), prospective coding.





# Head direction cells

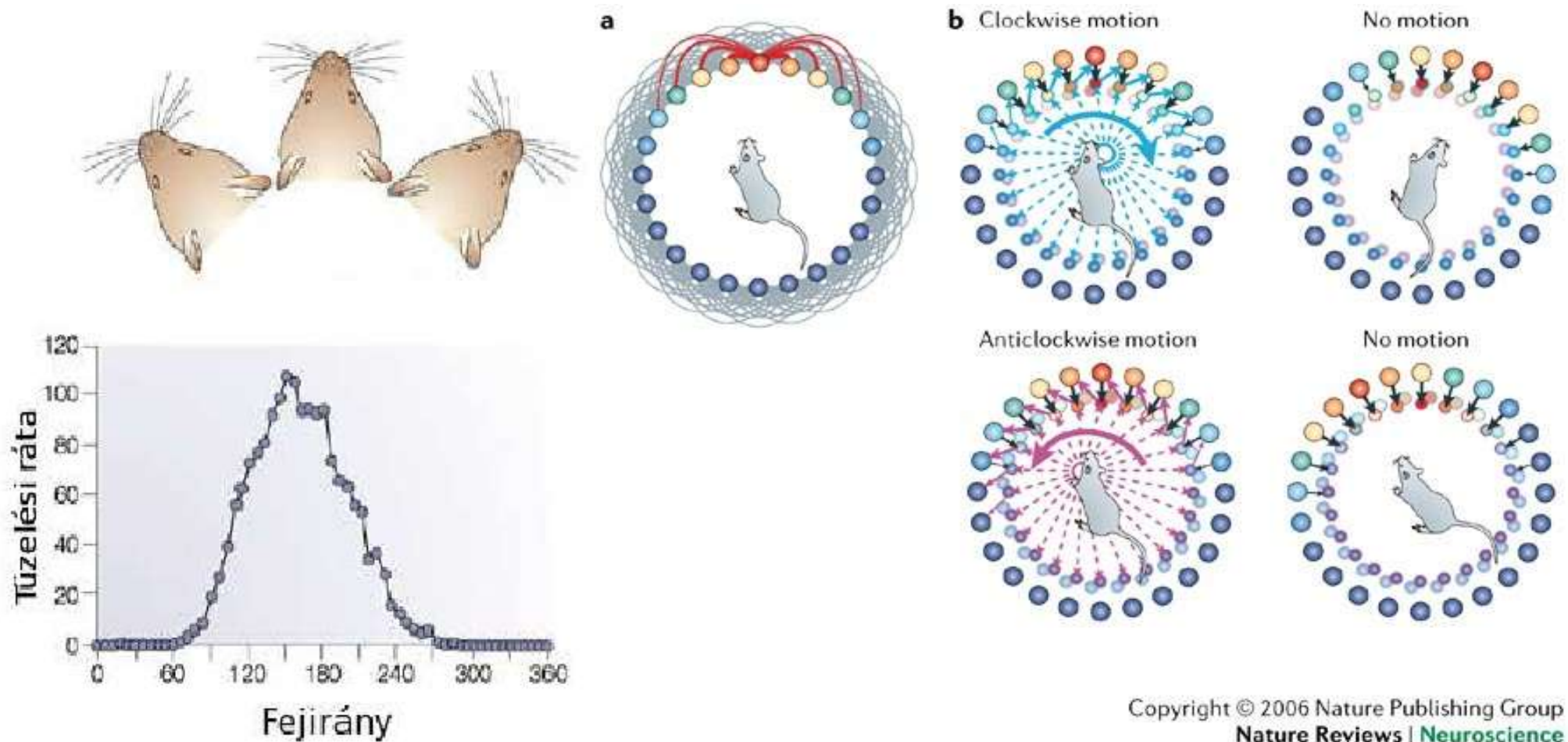
- Primary, based on external clues. (eq. visual)
- Without this, they are able to keep on the pattern, based on internal information (eq. vestibular)
- Removing the clue changes the firing pattern of all HD cells together by a random angle
- Firing of place cells change accordingly





# Head direction cells

An attractor network model: ring of excitatory and inhibitory connections



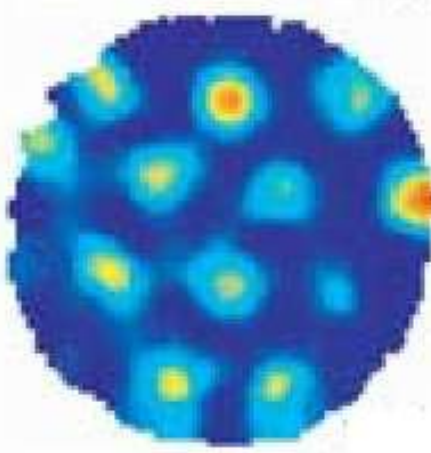
# Grid cells

- Pyramidal cells in the 2<sup>nd</sup> layer of the medial entorhinal cortex
- Position dependent activity
- Maximal firing frequency in the vertices of a triangular grid
- Parameters: spatial frequency, orientation, 2D phase

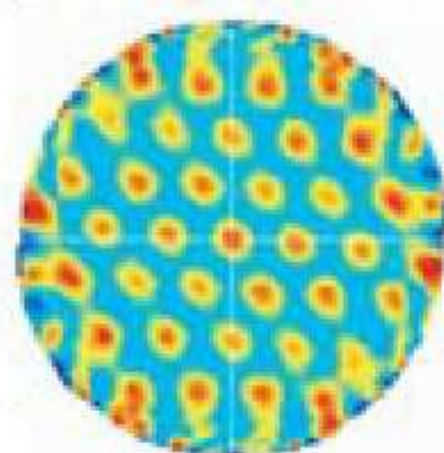
**action potentials**



**firing frequency**

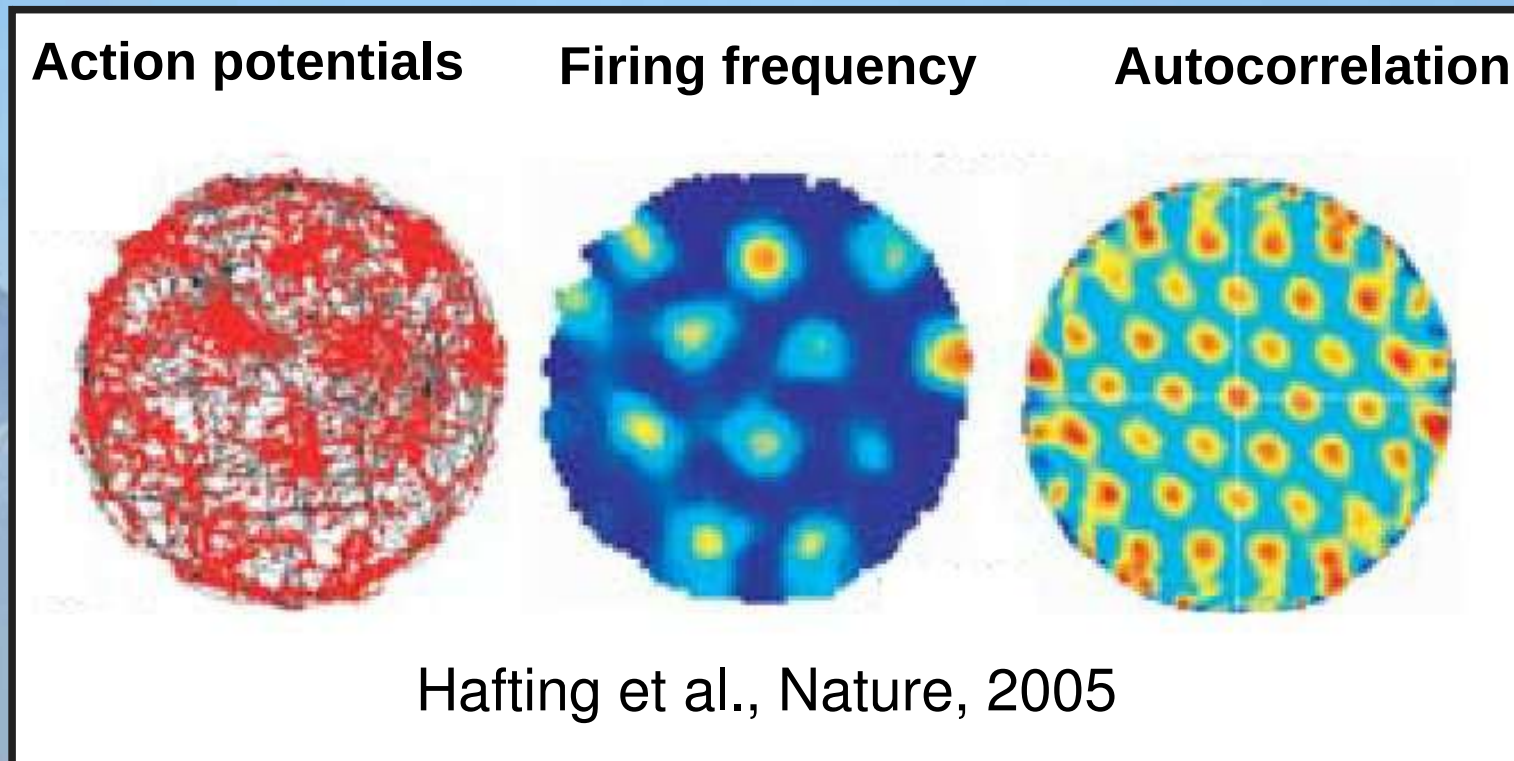


**autocorrelation**



Hafting et al., Nature, 2005

# A new type of spatial representation: The grid cells



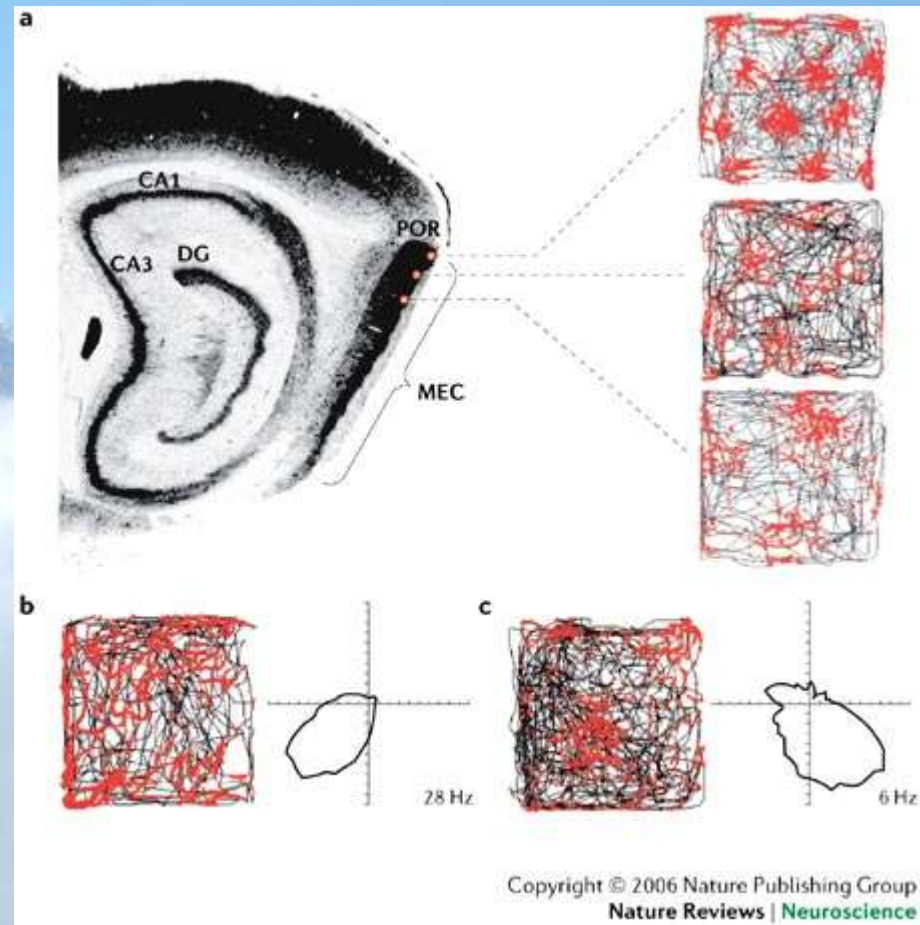
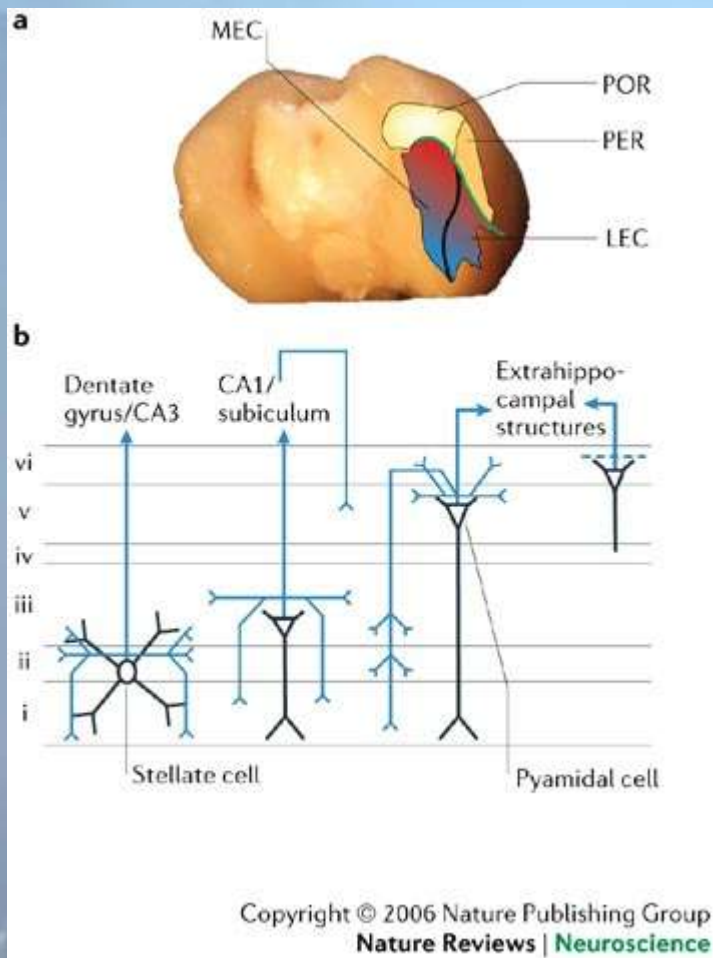
- Pyramidal cells in the 2<sup>nd</sup> layer of the medial entorhinal cortex
- Position dependent activity
- Maximal firing frequency in the vertices of a triangular grid
- Parameters: spatial frequency, orientation, 2D phase

**How does a crystal-lattice get to the brain?**



# The basic properties of the grid system

## Location and the structure of the MEC

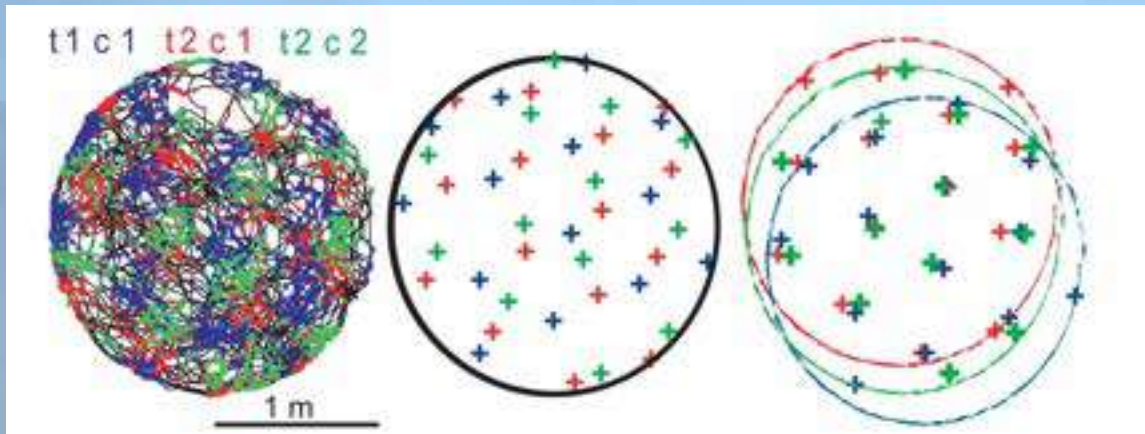


Periodicity

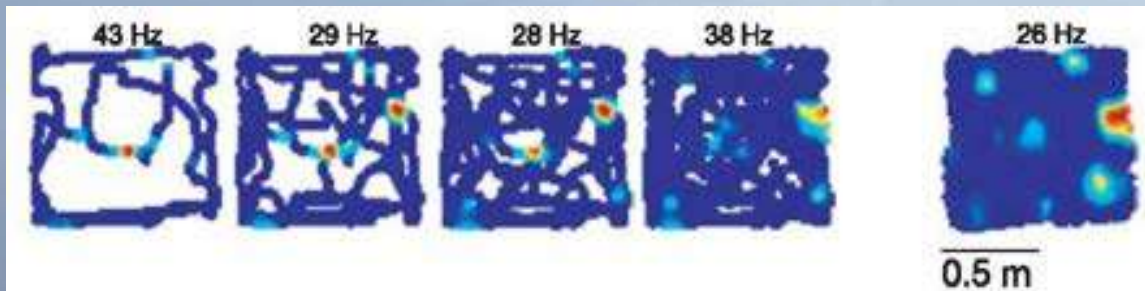
Grid periodicity increases along the dorso-ventral axis

# The basic properties of the grid system

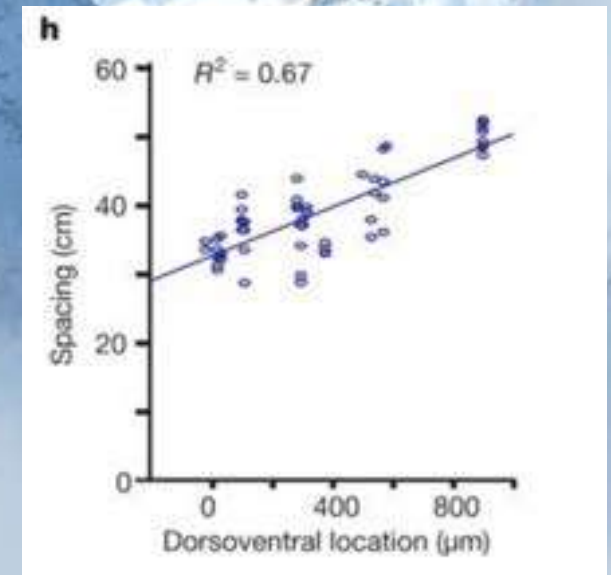
Phase: The neighboring cells have common period length, but their grid is shifted.



The grid pattern appears for the first run

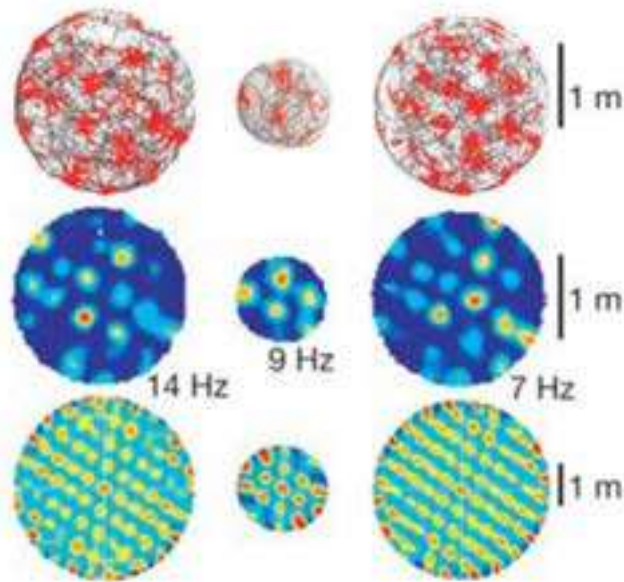


Periodicity

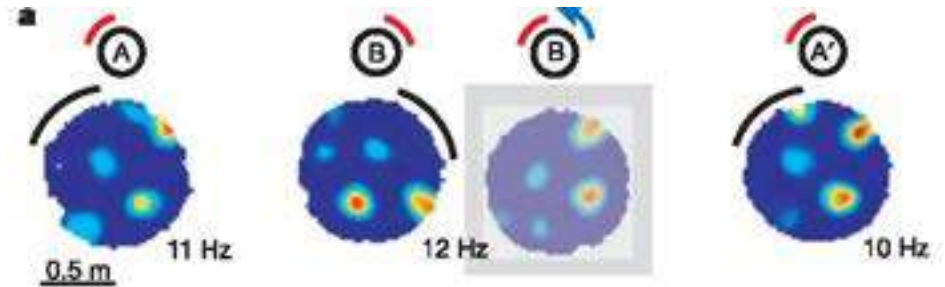
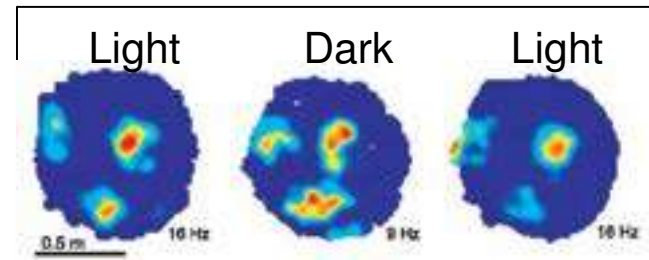


# Firing properties of the grid cells

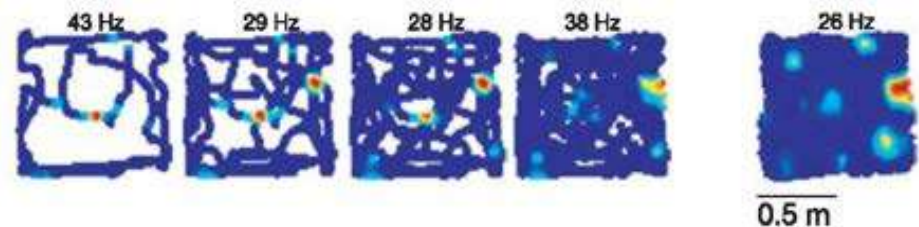
Independent of the size of the environment



Determined by visual clues



Immediate appearance



Fyhn et al., 2004  
Hafting et al., 2005



# Nobel prize in medicine 2014



## The 2014 Nobel Prize in Physiology or Medicine



**John O'Keefe**  
Born 1939, USA  
University College London



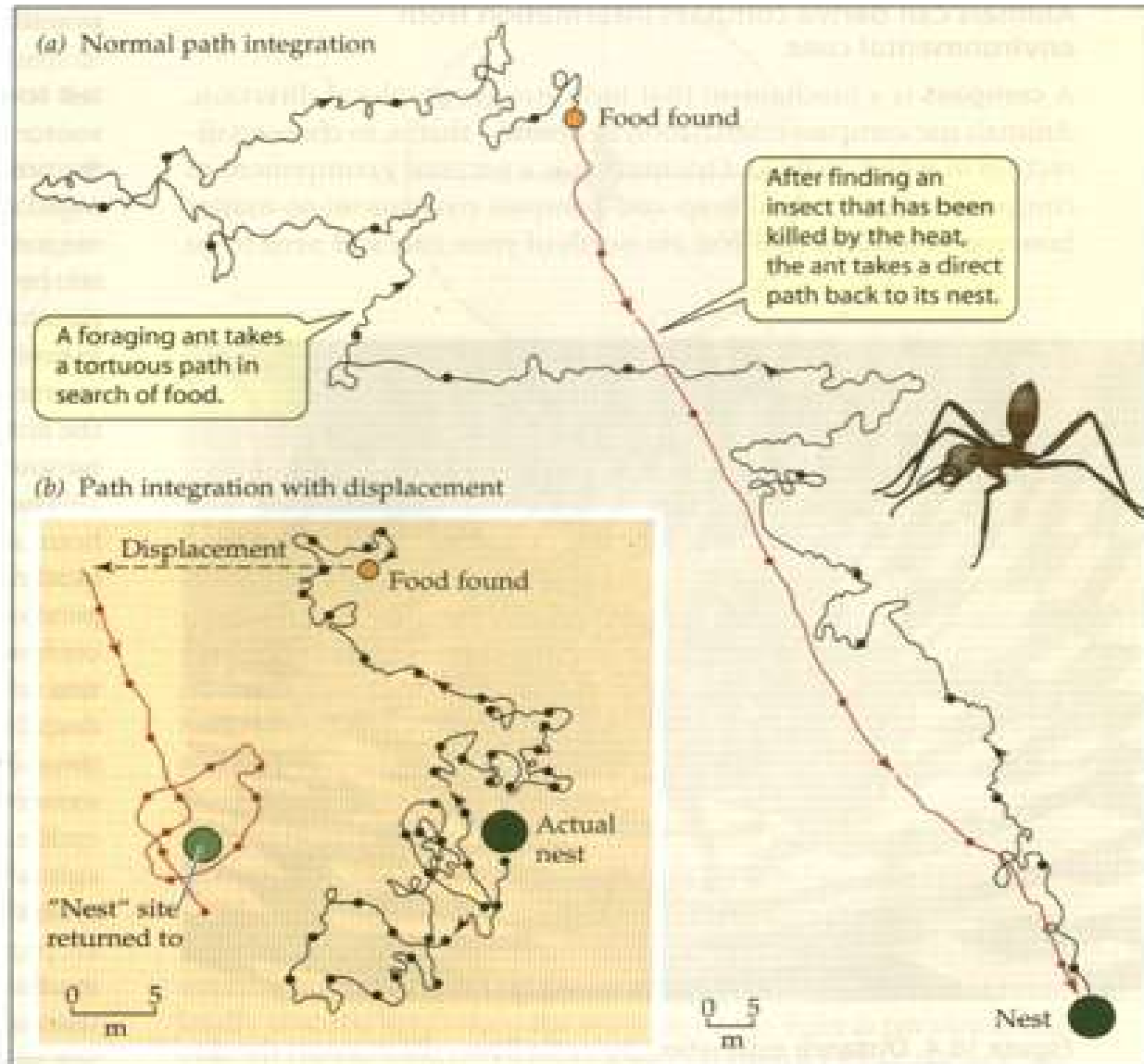
**May-Britt Moser**  
Born 1963, Norway  
Norwegian University  
of Science and  
Technology, Trondheim



**Edvard I. Moser**  
Born 1962, Norway  
Norwegian University  
of Science and  
Technology, Trondheim



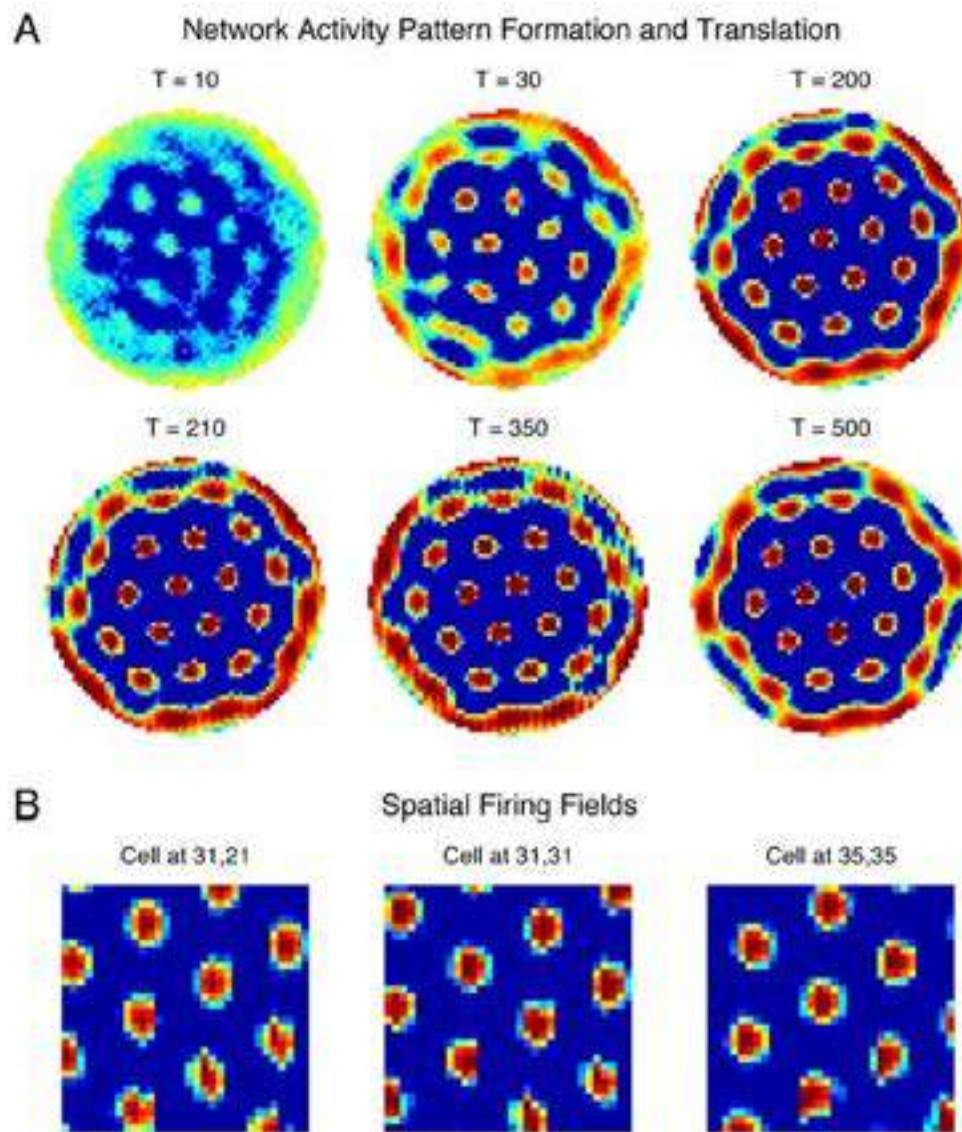
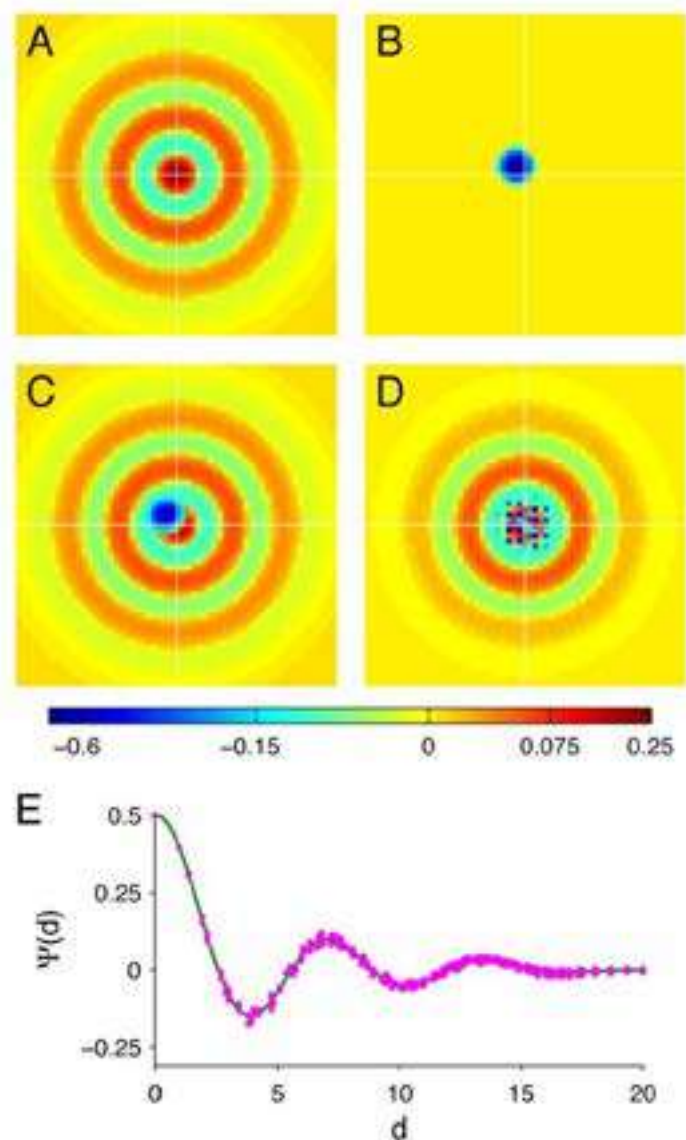
# Possible role: path integration





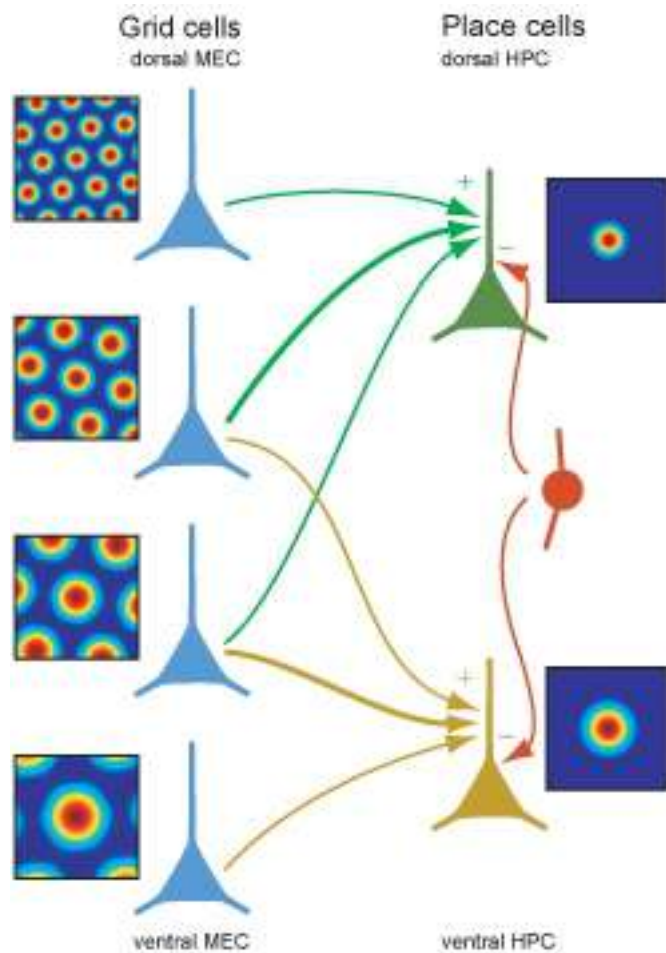
# A Spin Glass Model of Path Integration in Rat Medial Entorhinal Cortex

Mark C. Fuhs and David S. Touretzky (Journal of Neuroscience)



# How it is possible to determine the position based on grid code?

By summing up grids with corresponding phase



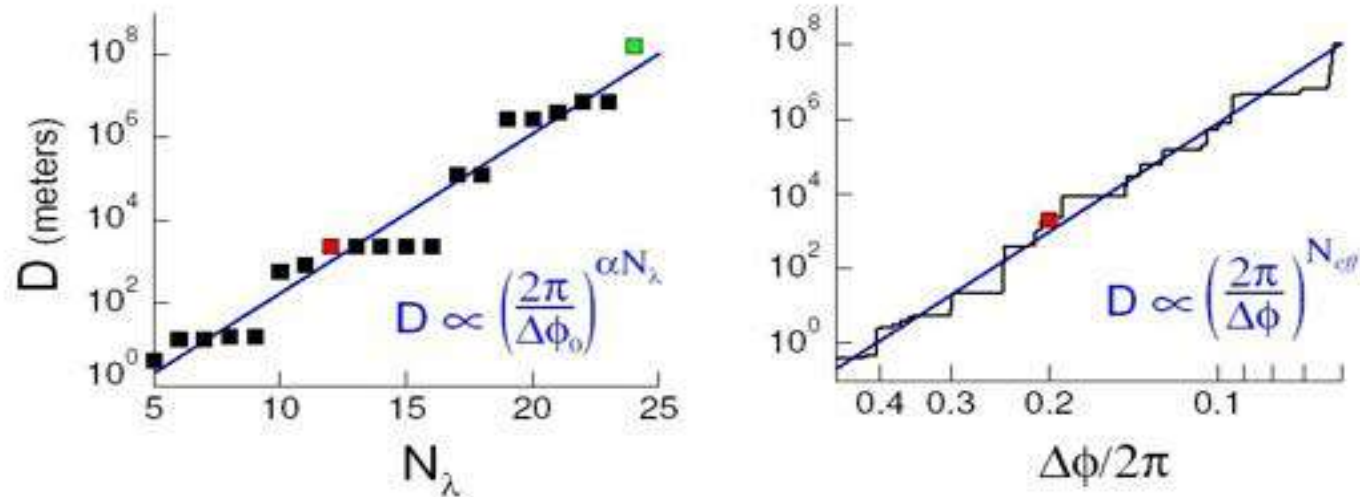
**LCM=185725**

	<i>Base numbers</i>			
	<i>17</i>	<i>19</i>	<i>23</i>	<i>25</i>
<i>Number</i>	<i>Residum</i>			
<i>3246</i>	<i>16</i>	<i>16</i>	<i>3</i>	<i>21</i>
<i>3247</i>	<i>0</i>	<i>17</i>	<i>4</i>	<i>22</i>
<i>3248</i>	<i>1</i>	<i>18</i>	<i>5</i>	<i>23</i>

This corresponds to a Chinese remainder numeral system. Unique until the least common multiple.

# The capacity increases exponentially

Yoram Burak, Ted Brookings, Ila Fiete (arXiv)



$N_\lambda$	$\Delta\phi_0/2\pi$	$D$ (m)	# grid cells	# place cells
12	0.2	$2 \times 10^3$	$5 \times 10^4$	$\sim 10^{10}$
24	0.2	$2 \times 10^8$	$1 \times 10^5$	$\sim 10^{20}$

# Models

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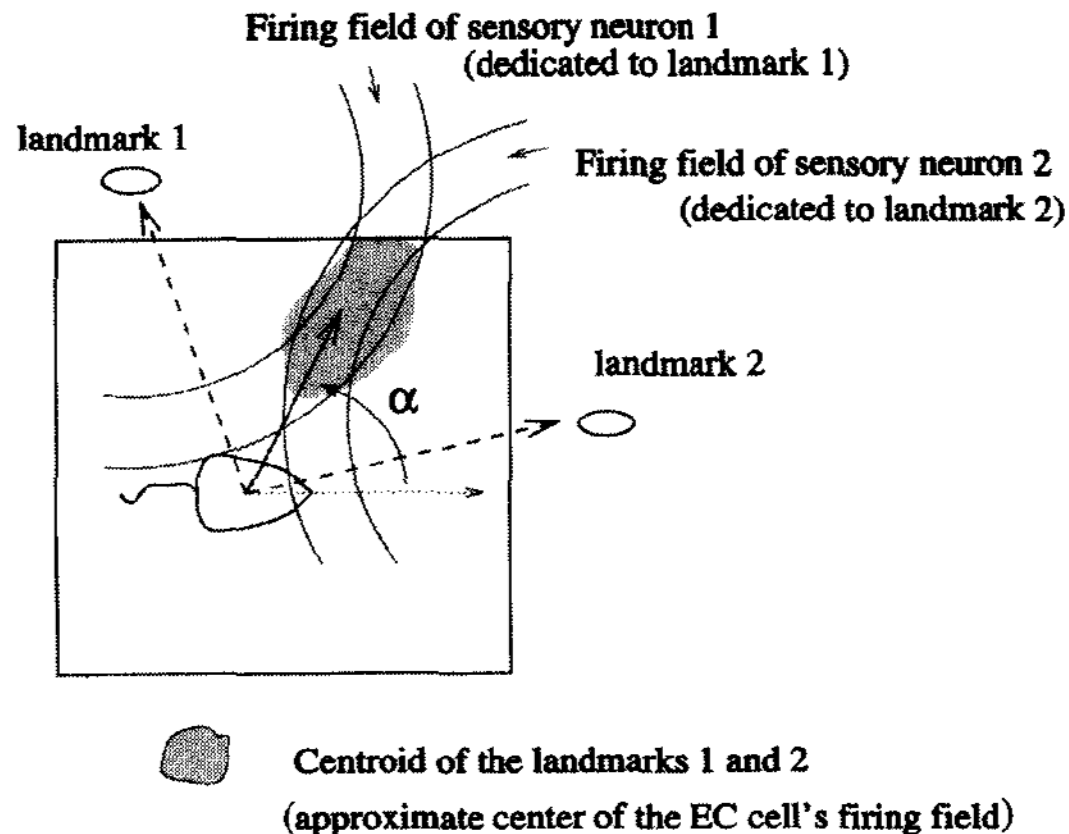


Fig. 24. Firing phase computation yields  $\alpha$ , the angle between the heading direction of the animal and the direction defined by the position of the animal and the centroid of landmarks 1 and 2; the phase will be "Late" if  $|\alpha|$  is smaller than  $60^\circ$ , "Middle" if  $|\alpha|$  is between  $60^\circ$  and  $120^\circ$  and "Early" if  $|\alpha|$  is greater than  $120^\circ$ . (After Burgess *et al.*, 1994.)

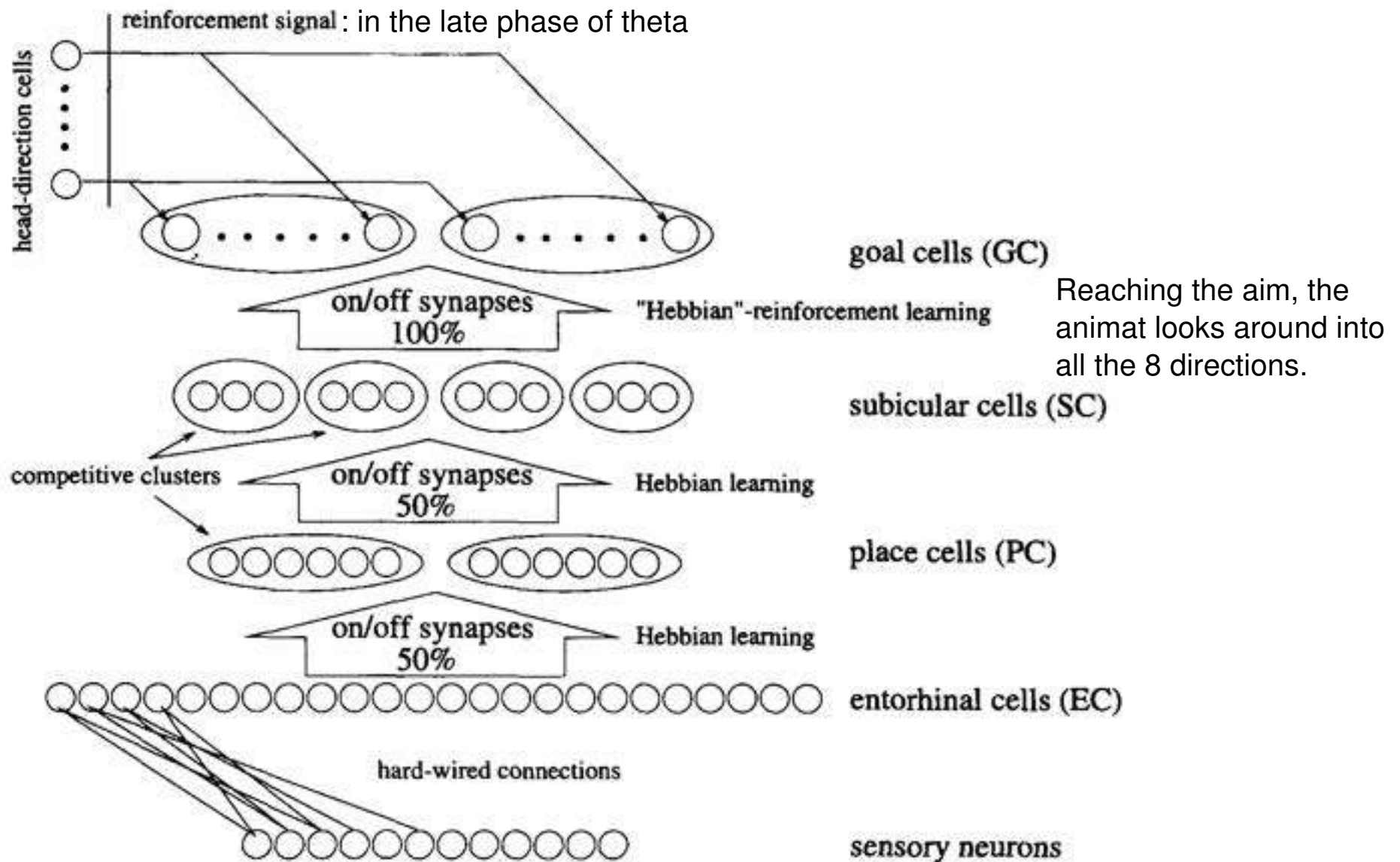


Fig. 23. Burgess *et al.*'s feedforward network inspired by the hippocampus architecture. Cells (circles) in some layers are organized into clusters (ellipses). There are five clusters of 50 place cells, 10 groups of 25 subicular cells and eight goal cells for each goal, corresponding to 8 head-direction cells. (After Burgess *et al.*, 1994.)

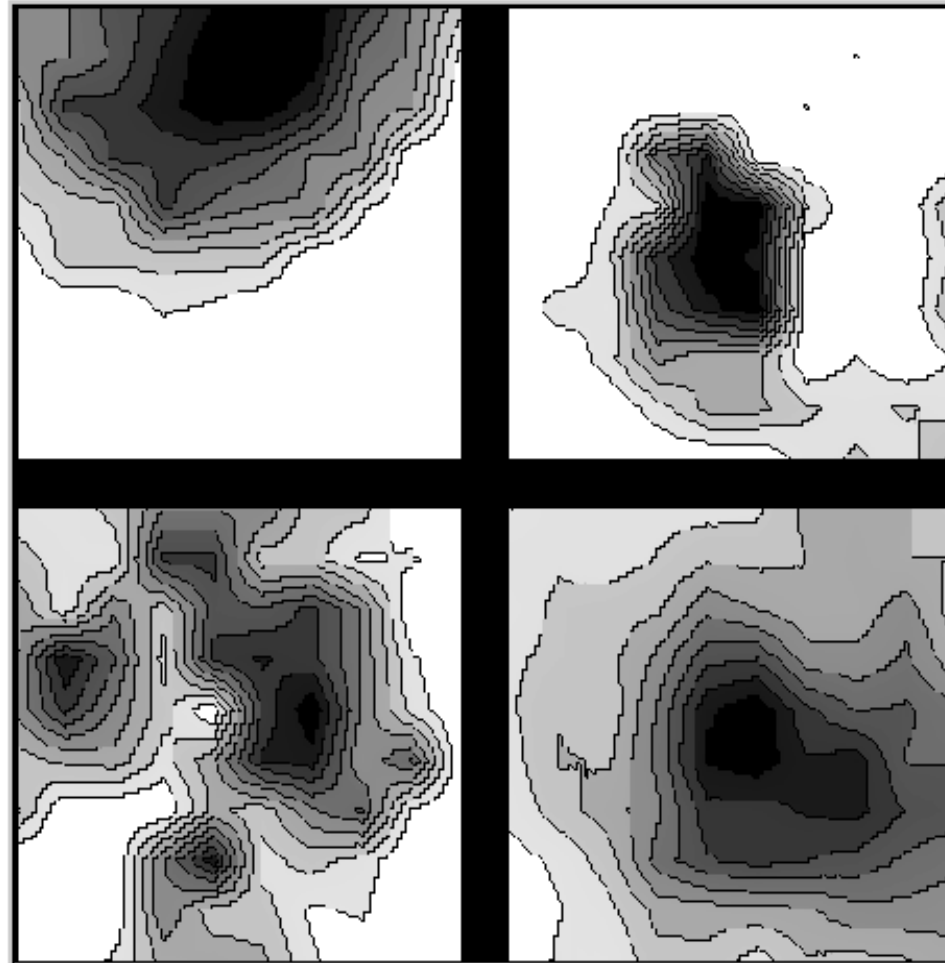


Figure 8: Typical firing rate maps of cells in the different layers of the model after 60s exploration, showing qualitative agreement with known extracellular recordings from entorhinal cells and place cells, and predicting firing rate maps for subicular cells and 'goal cells'. The simulated rat moves evenly across the environment; spikes were binned in a  $10 \times 10$  grid, each contour represents 10% of the peak firing rate. Top row: entorhinal cell, peak rate is 40Hz (left), place cell peak rate is 30Hz, (right). Bottom row: subicular cell, peak rate is 40Hz (left), goal cell representing east (the goal is at the centre), peak rate 101Hz (right).

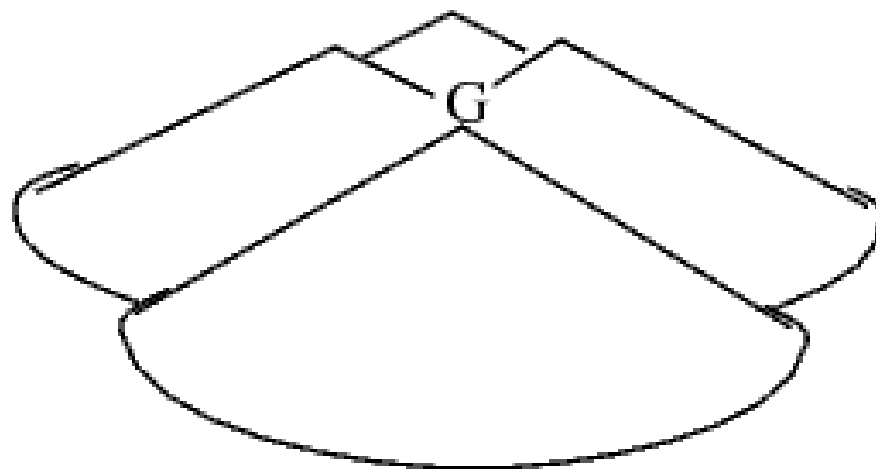


Figure 5: One possible set of firing rate maps giving rise to a population vector representing the position of the rat from the goal at G.



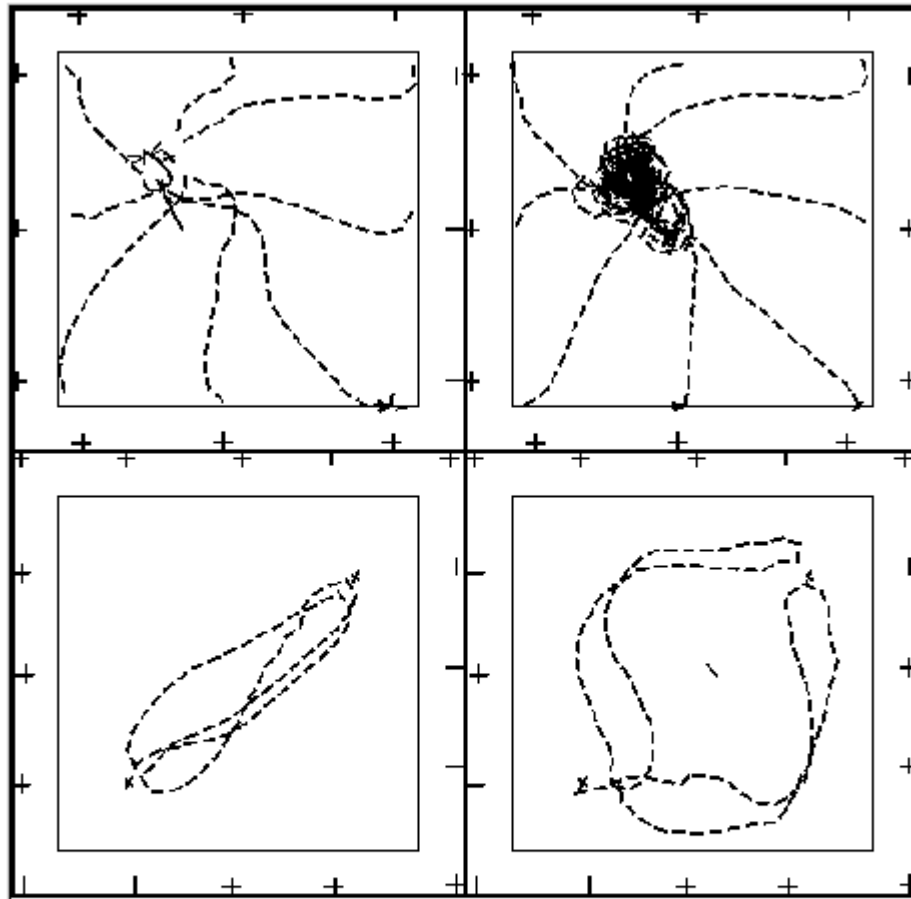


Figure 9: Trajectories taken by the simulated rat. Goals (marked by 'x') were encountered after 30s exploration in the goal-less environment. Each dash represents one movement (0.1s), cues are marked by '+'. The average initial number of on connections to a PC or SC,  $\langle C_{in} \rangle$ , was 1. Top: navigating to the goal from 8 starting positions on the first block of trials after encountering the goal, average escape latency is 1.4s (left), navigation with the goal removed, showing localisation of search (right). Bottom: navigation between two goals (left), and after an obstacle (marked by '\') is encountered (right).

## Summary

- One-shot learning at goal
- Hebbian learning
- Latent learning
- Phase-precession
- Sensory neurons?
- Goal cells?
- PC and SC layers are unnecessary
- Works only in a limited distance from the goal (SC place field size)

# Reinforcement learning: an actor-critique architecture

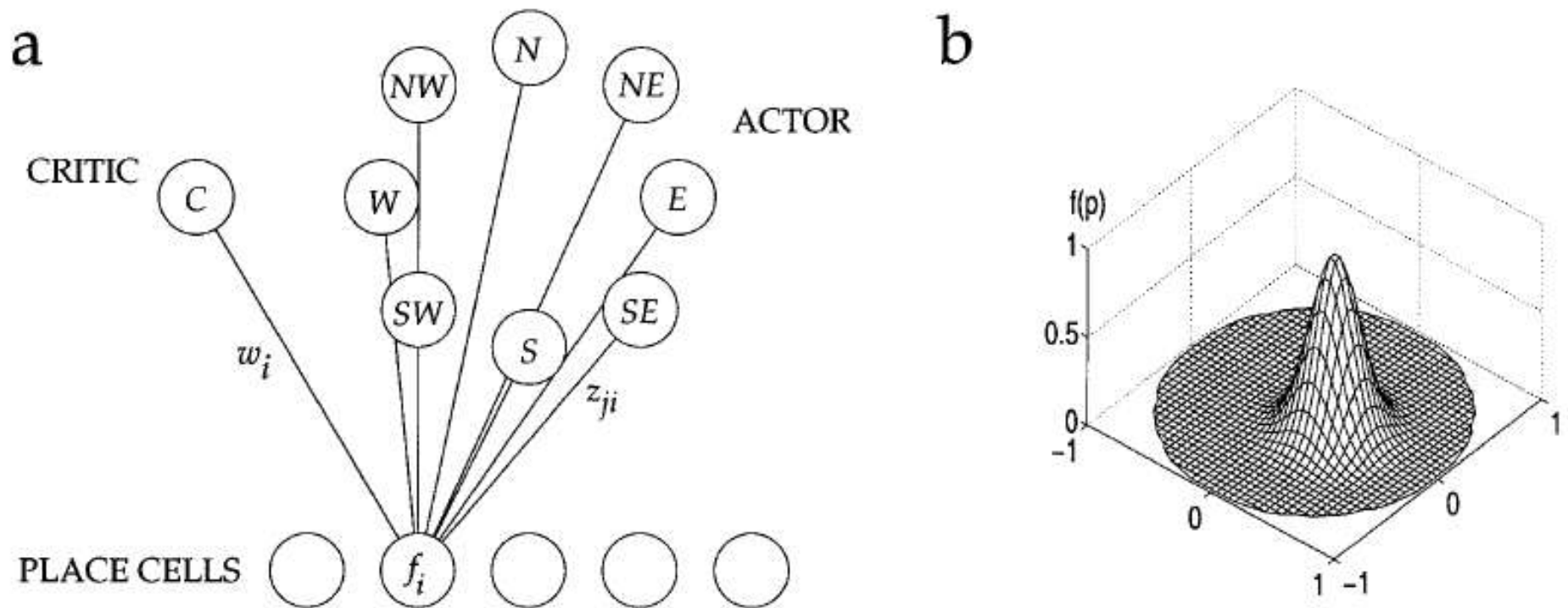
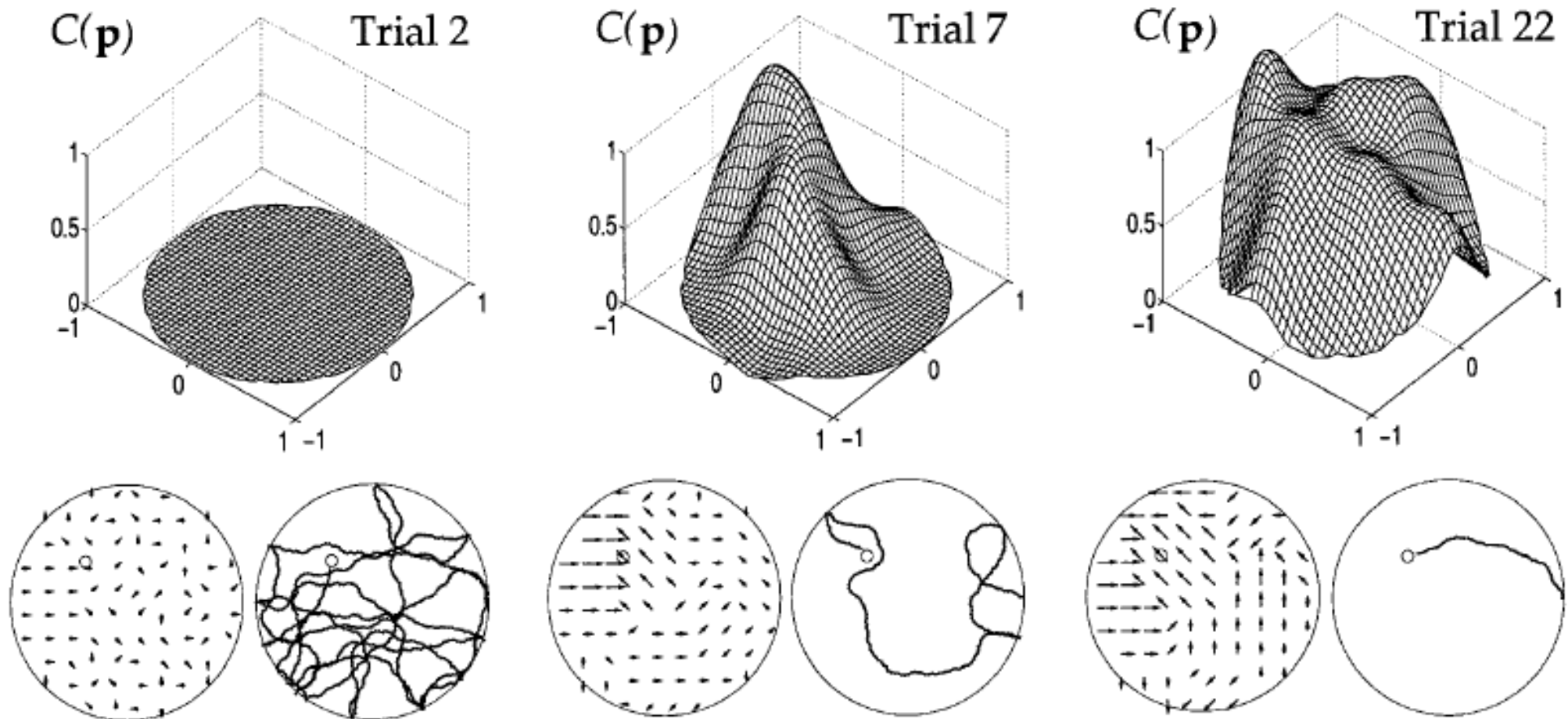


FIGURE 2. The actor-critic system. a: An input layer of place cells projects to the critic cell,  $C$ , whose output is used to evaluate behavior. Place cells also project to eight action cells, which the actor

uses to select between eight possible directions of movement from any given location. b: An example of a Gaussian place field (x and y axes represent location, z axis represents firing rate).

# Parallel learning of policies and the values



# The temporal difference rule:

The critic depends on position  $p$ :

$$C(p) = \sum_i w_i f_i(p)$$

The value function ( $V$ ) must satisfy:

$$V(p_t) = \langle R_t + \gamma R_{t+1} + \gamma^2 R_{t+2} + \dots \rangle$$

Where  $\gamma$  is a constant discount factor for predicted (not actual) reward. From this the consistency of the value:

$$V(p_t) = \langle R_t \rangle + \gamma V(p_{t+1}).$$

A well trained critic should satisfy the same consistency assumption:

$$C(p_t) = \langle R_t \rangle + \gamma C(p_{t+1}).$$

The actual difference between the two sides governs the learning, this is called the temporal difference learning rule:

$$\delta_t = R_t + \gamma C(p_{t+1}) - C(p_t)$$

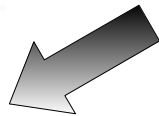
The weight changes are proportional to the difference :

$$\Delta w_i \propto \delta_t f_i(p_t).$$

# The temporal difference rule:

Two sources of value:

$$\delta_t = R_t + \gamma C(p_{t+1}) - C(p_t)$$



The actually achieved  
reward



The difference between the  
expected and the achieved  
increase.

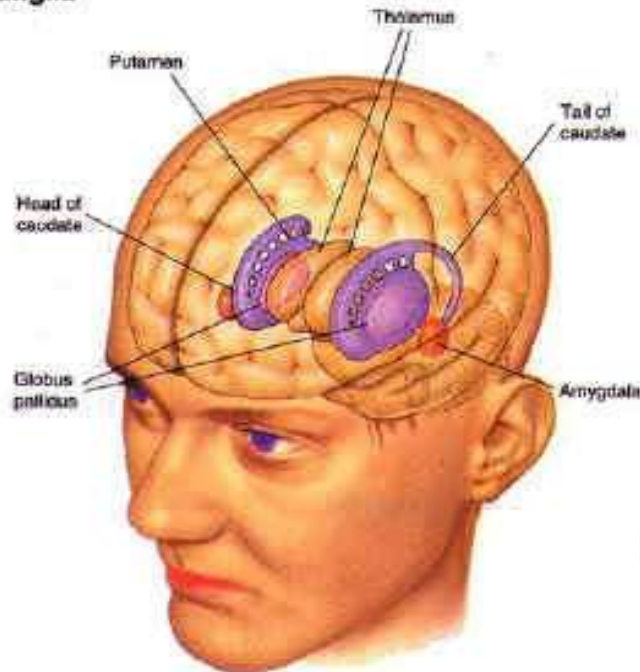
Together expresses the difference between the expected  
and actual reward.

# The effect of reward in dopaminergic cell of basal ganglia

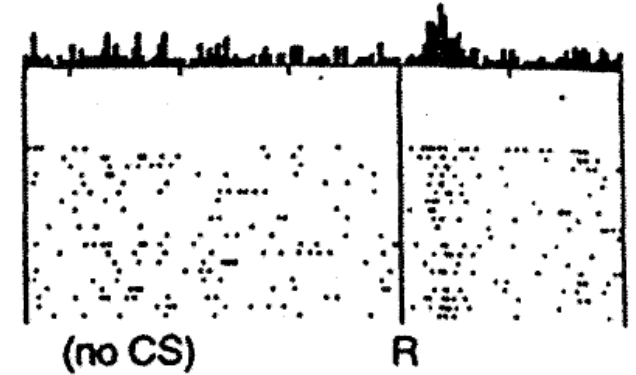
An interpretation:

Dopamine cells signal the difference between the expected and received reward.

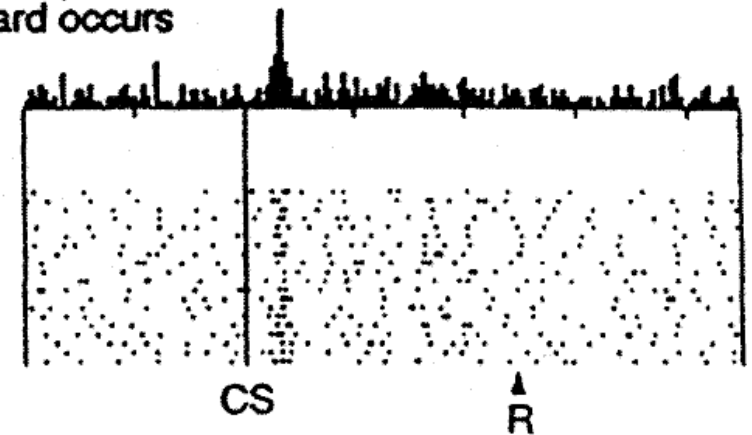
► The Basal Ganglia



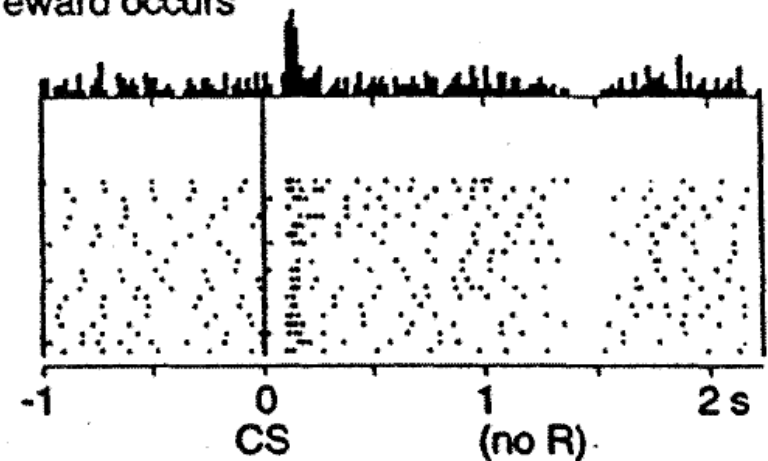
No prediction  
Reward occurs



Reward predicted  
Reward occurs



Reward predicted  
No reward occurs



# Animat navigation using a cognitive graph

Olivier Trullier, Jean-Arcady Meyer: *Biol. Cybern.* 2000

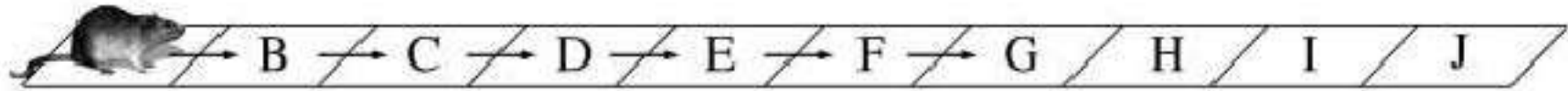
This article describes a computational model of the hippocampus that makes it possible for a simulated rat to navigate in a continuous environment containing obstacles. This model views the hippocampus as a ``cognitive graph'', that is, a hetero-associative network that learns temporal sequences of visited places and stores a topological representation of the environment.

Calling upon place cells, head direction cells, and ``goal cells'', it suggests a biologically plausible way of exploiting such a spatial representation for navigation that does not require complicated graph-search algorithms. Moreover, it permits ``latent learning'' during exploration.

The model implements a simple ``place-recognition-triggered response'' navigation strategy. It implements and uses fine details as phase precession and spike time dependent plasticity.

# Animat navigation using a cognitive graph

Olivier Trullier, Jean-Arcady Meyer: *Biol. Cybern.* 2000



A → B → C → D → E → F → G  
B → C → D → E → F → G → H  
C → D → E → F → G → H → I  
...

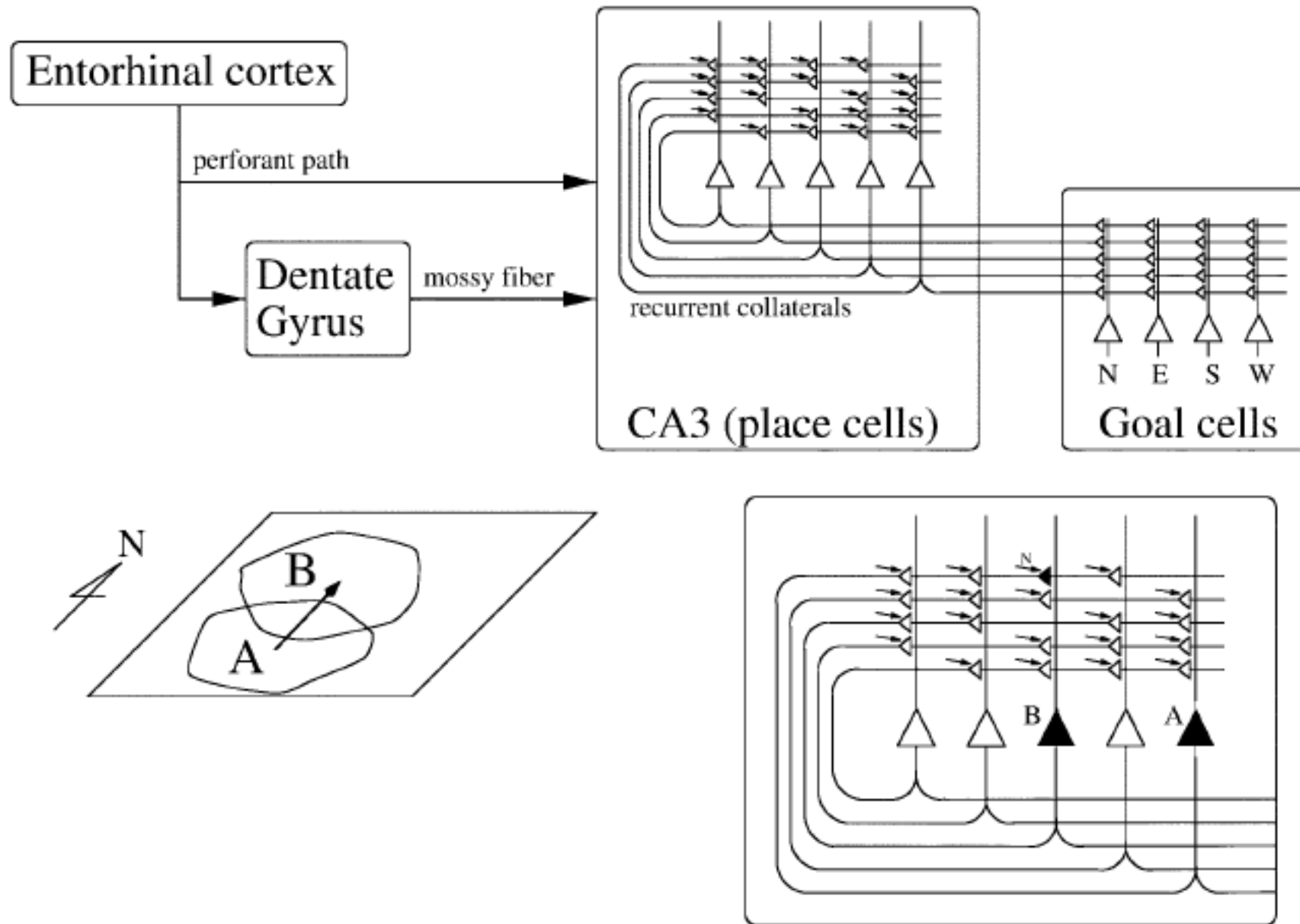
Phase precession explained by sequence learning. Each bin in this grid-like world corresponds to a unique place. The rat has learned the sequence of places from A to J. It subsequently moves from A to J

(top): when it is in A, it recalls the sequence from A to G (bottom); when it is in B, it recalls the sequence from B to H; and so on. Each movement and each prediction phase takes a full theta cycle. Thus, the representation of the current place starts a new theta cycle and the prediction of place E comes earlier and earlier in the cycle (dotted arrow), that is the phase of @ring of the place cell corresponding to E diminishes



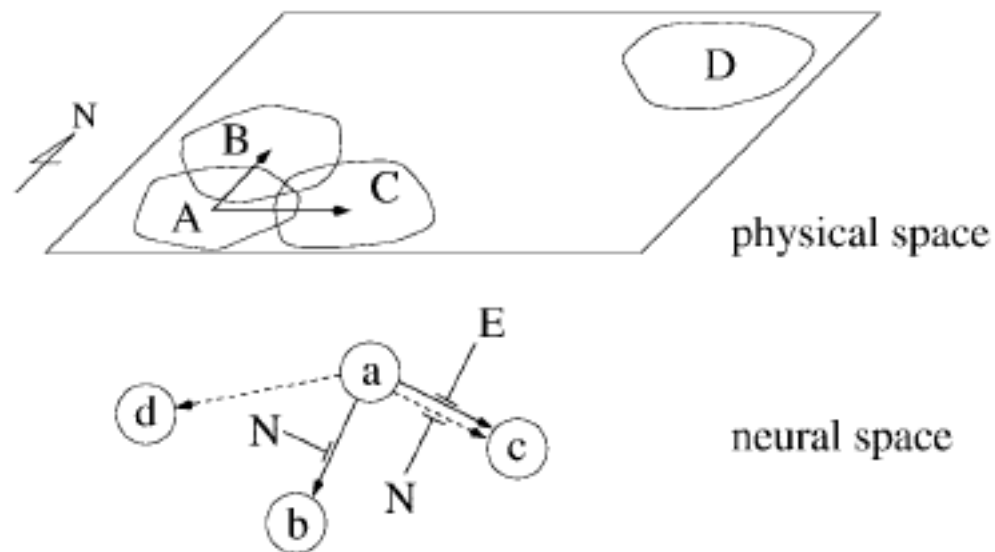
# The structure of the model

Olivier Trullier, Jean-Arcady Meyer: *Biol. Cybern.* 2000



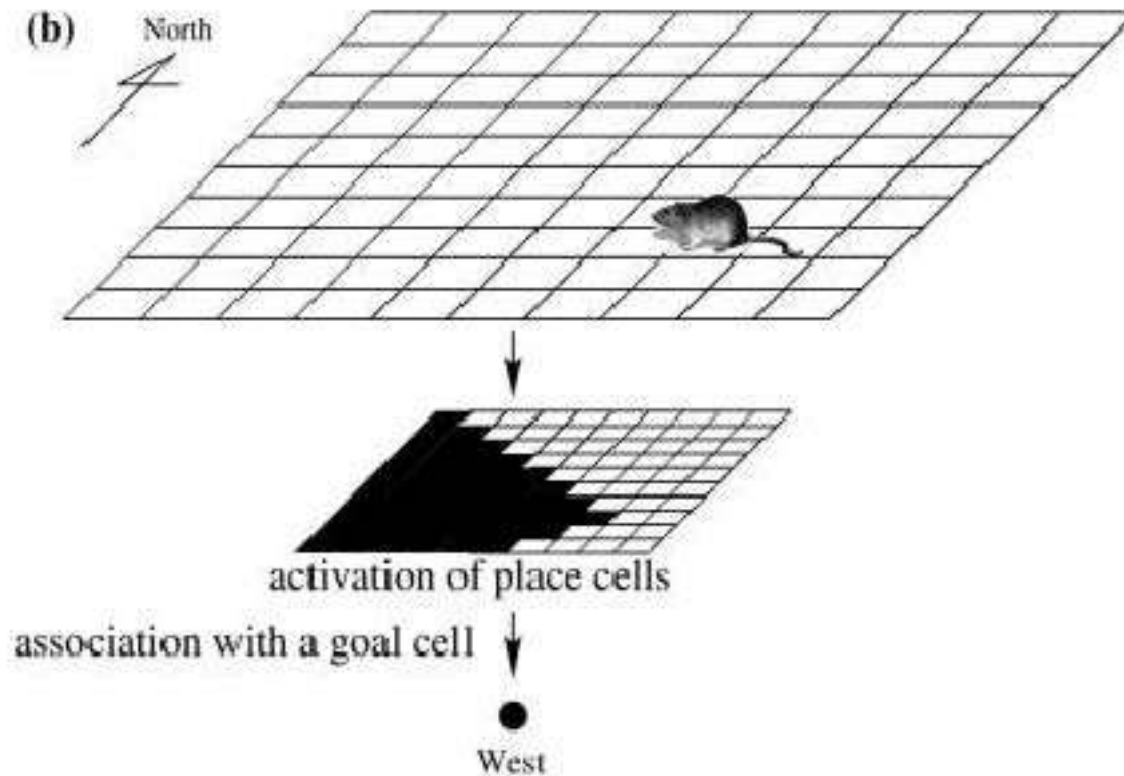
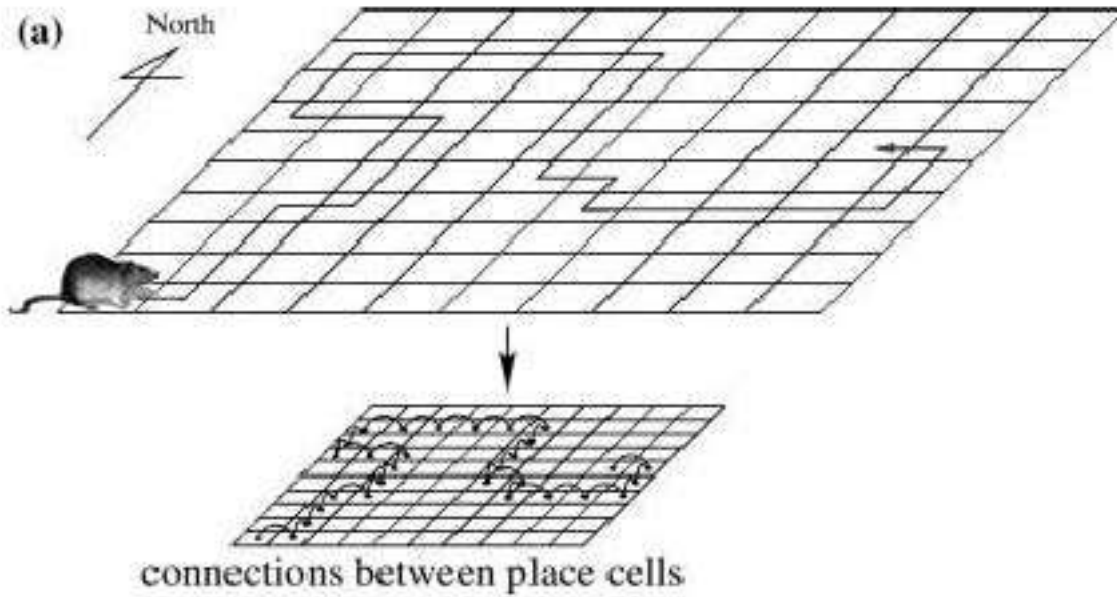
# Learning of cognitive graph

Olivier Trullier, Jean-Arcady Meyer: *Biol. Cybern.* 2000



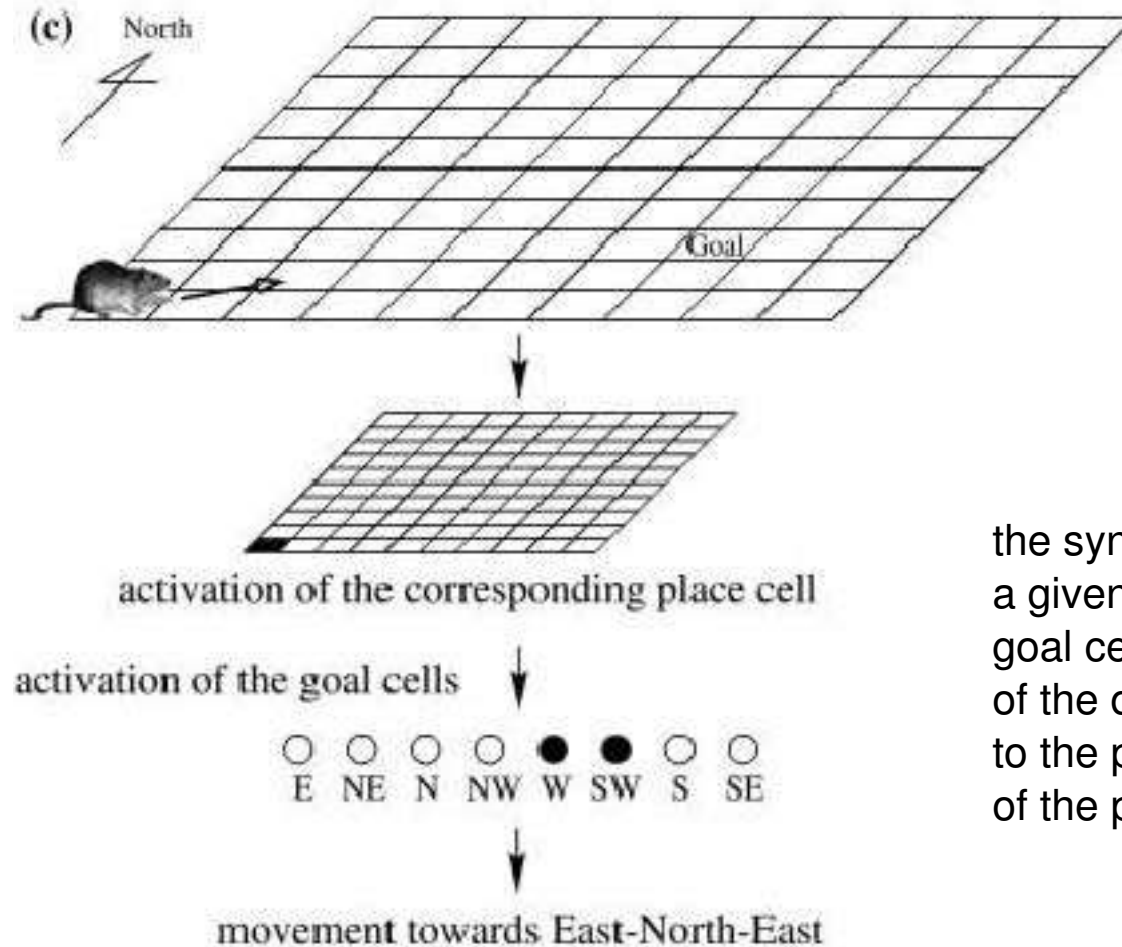
The modified connection between two place cells in neural space corresponds to the facts that the corresponding placefields are neighbors and that the place field of the post-synaptic cell is in the direction corresponding to the head-direction that modulates the connection, with respect to the place field of the pre-synaptic cell.

# Exploration



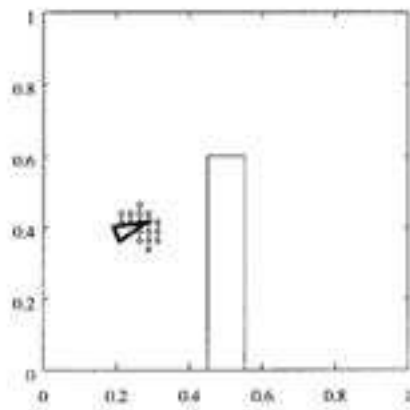
the synaptic weight between a given place cell and a given goal cell is an inverse function of the distance from the goal to the preferred location of the place cell.

# Exploitation

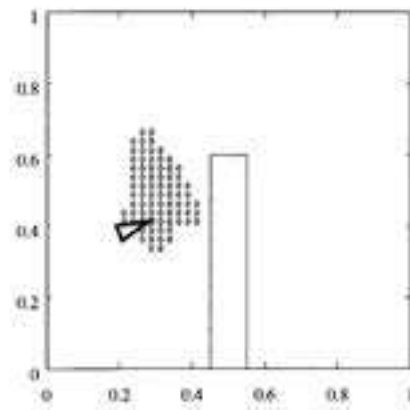


the synaptic weight between a given place cell and a given goal cell is an inverse function of the distance from the goal to the preferred location of the place cell.

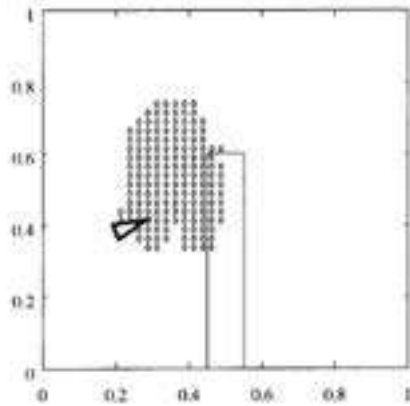
**Signal propagation  
from the goal  
location towards the  
east.**



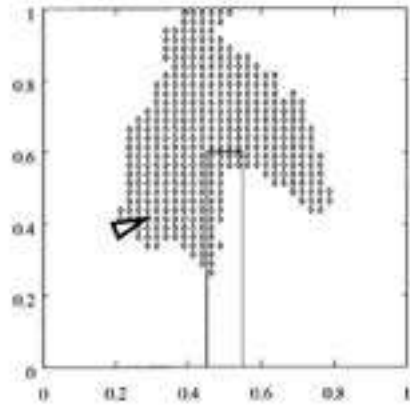
(a) initial state



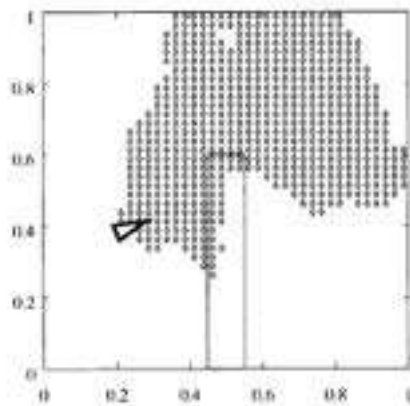
(b) after 5 iterations



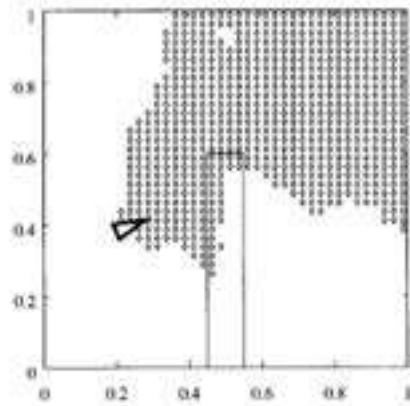
(c) after 10 iterations



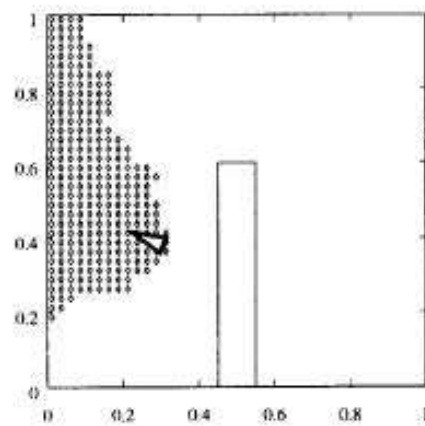
(d) after 20 iterations



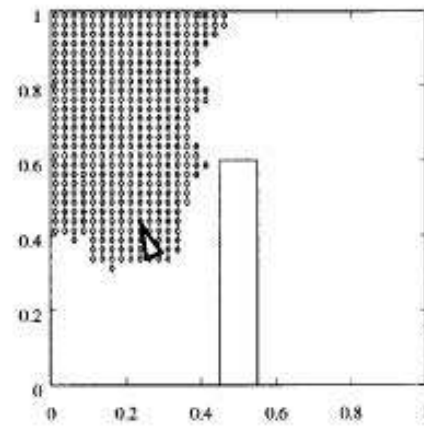
(e) after 30 iterations



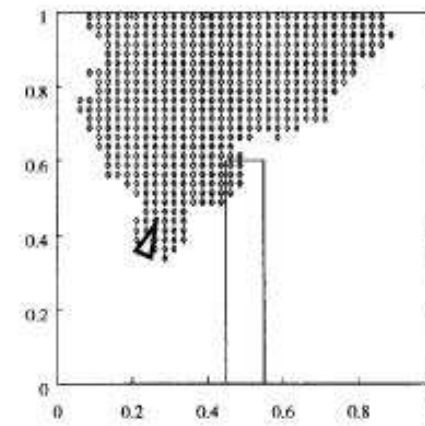
(f) after 38 iterations



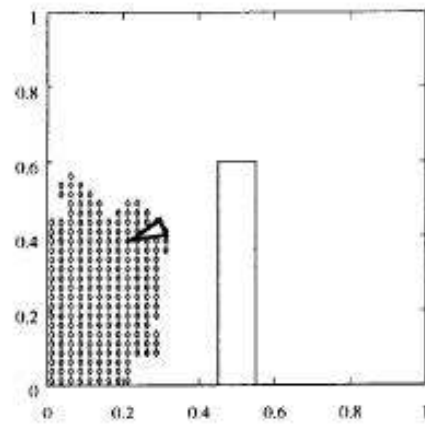
(a) North-West



(b) North

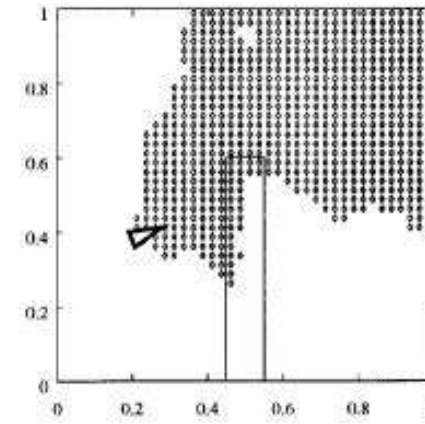


(c) North-East

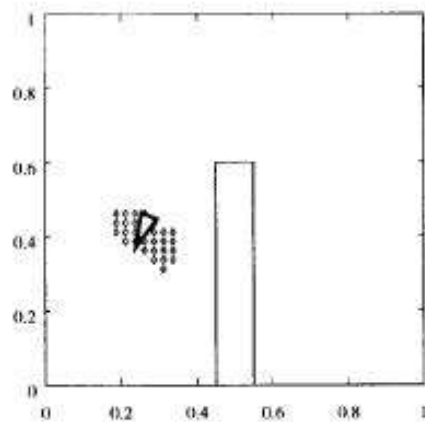


(d) West

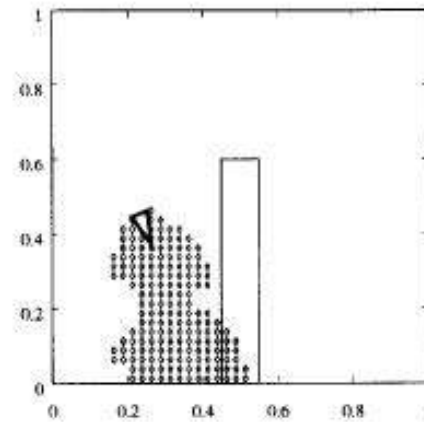
**Activity fields  
of the  
eight goal cells**



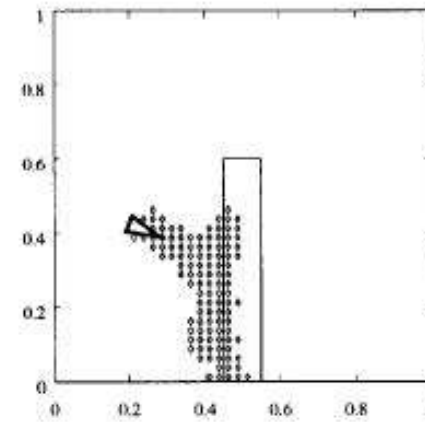
(e) East



(f) South-West



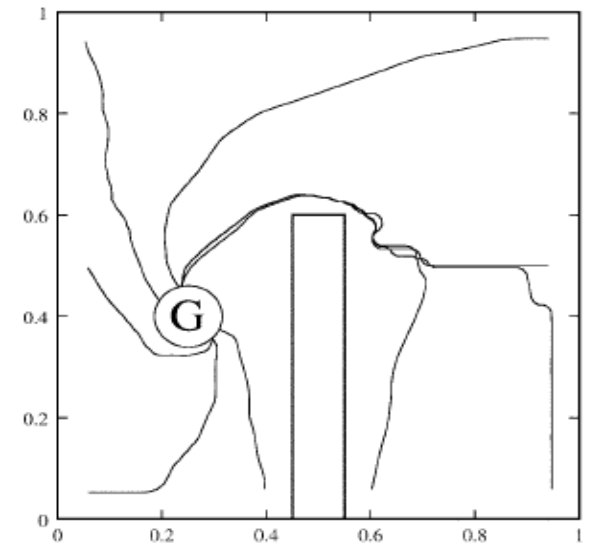
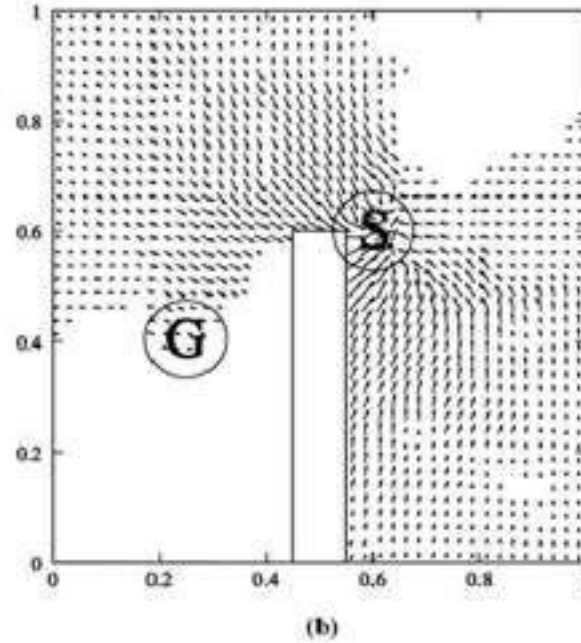
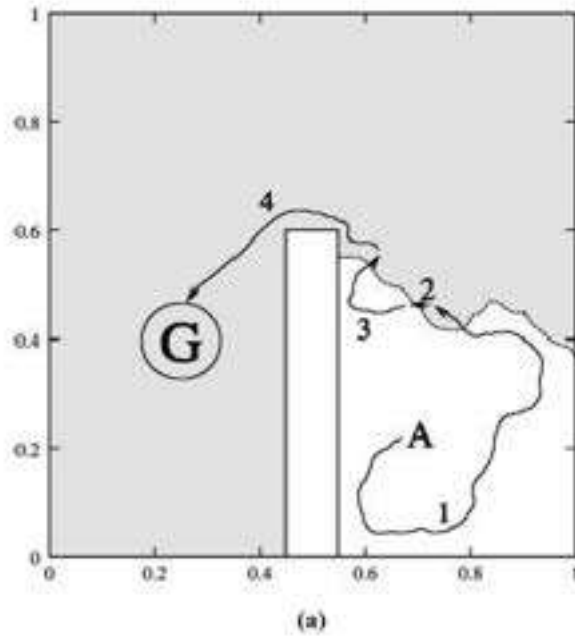
(g) South



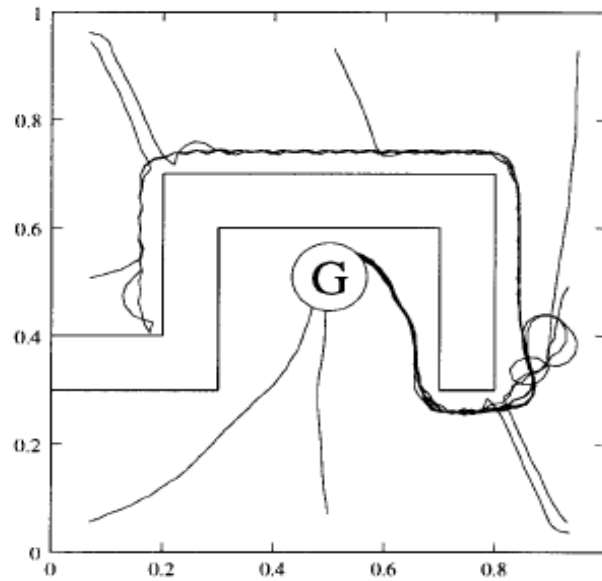
(h) South-East

# Creating subgoals

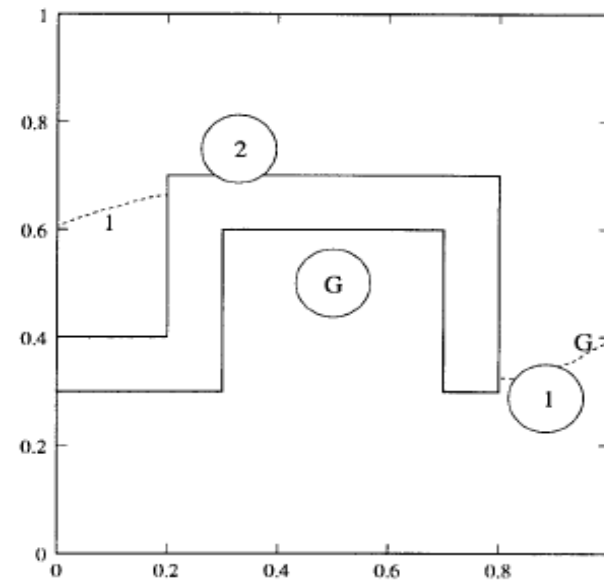
# Resulting trajectories



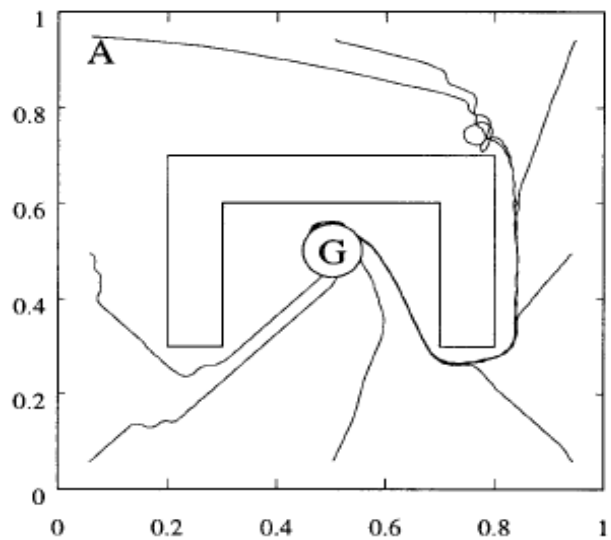
# Possible alternative routes



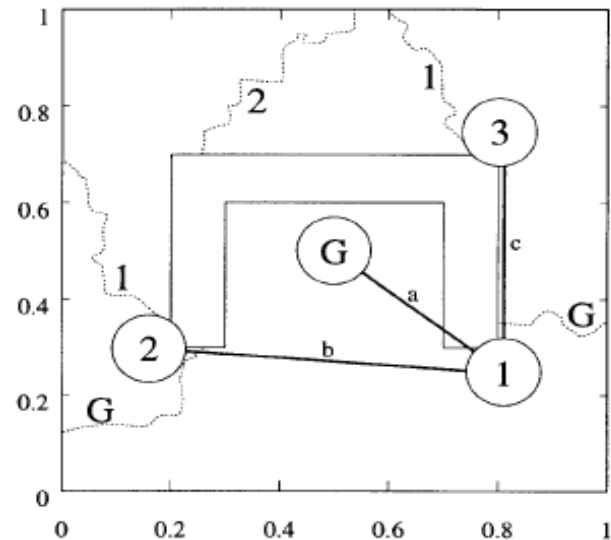
(a)



(b)



(c)



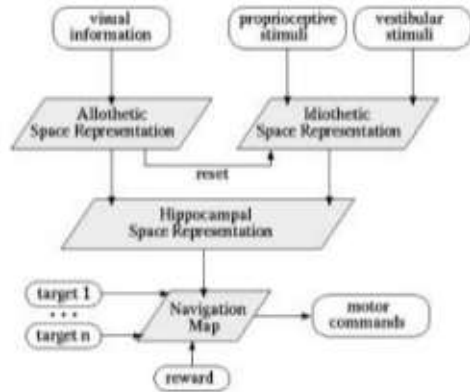
(d)



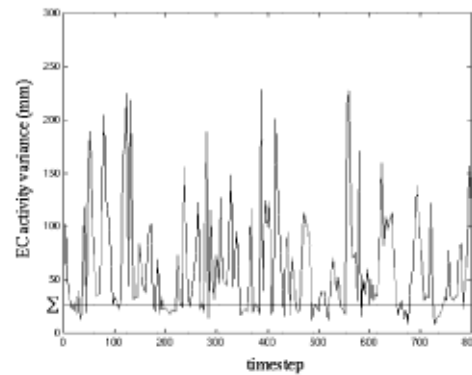
# Spatial cognition and neuro-mimetic navigation: a model of hippocampal place cell activity

Angelo Arleo, Wulfram Gerstner

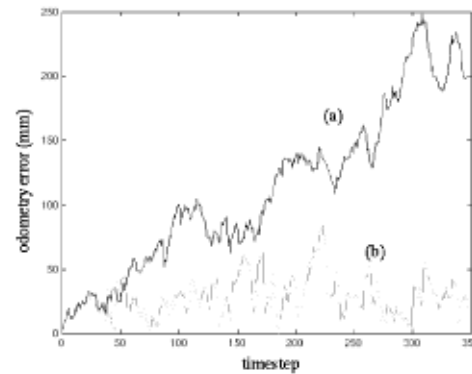
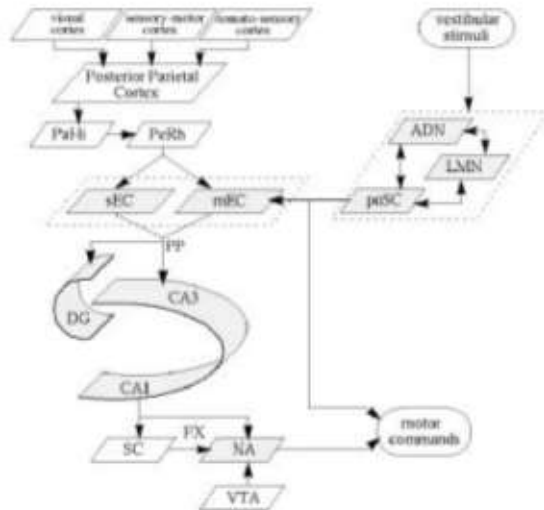
Centre for Neuro-Mimetic Systems, MANTRA, Swiss Federal Institute of Technology Lausanne, 1015 Lausanne, EPFL, Switzerland



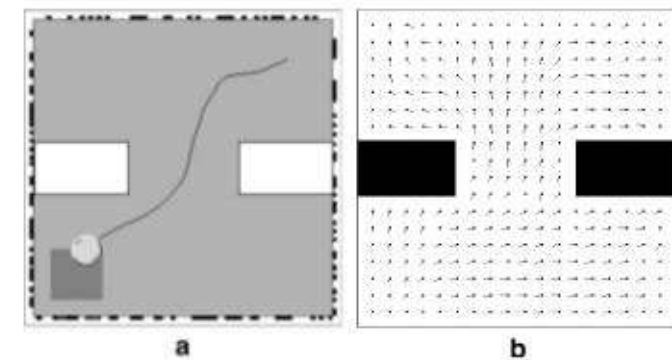
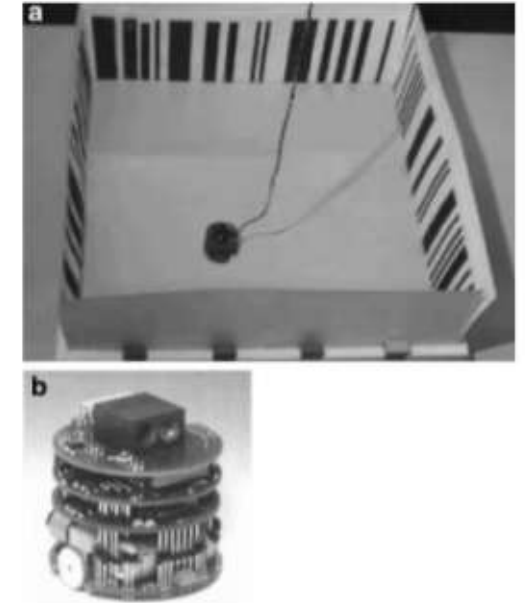
**Fig. 1.** Functional overview of the model. Allothetic and idiothetic stimuli are combined to yield the hippocampal space representation. Navigation is based on place cell activity, desired targets, and rewards



**Fig. 8.** The variance of the sEC cell activity around the center of mass  $\mathbf{p}_{\text{acc}}$ . When the variance falls below the fixed threshold  $\Sigma$  the spatial location  $\mathbf{p}_{\text{acc}}$  is used to calibrate the robot's position

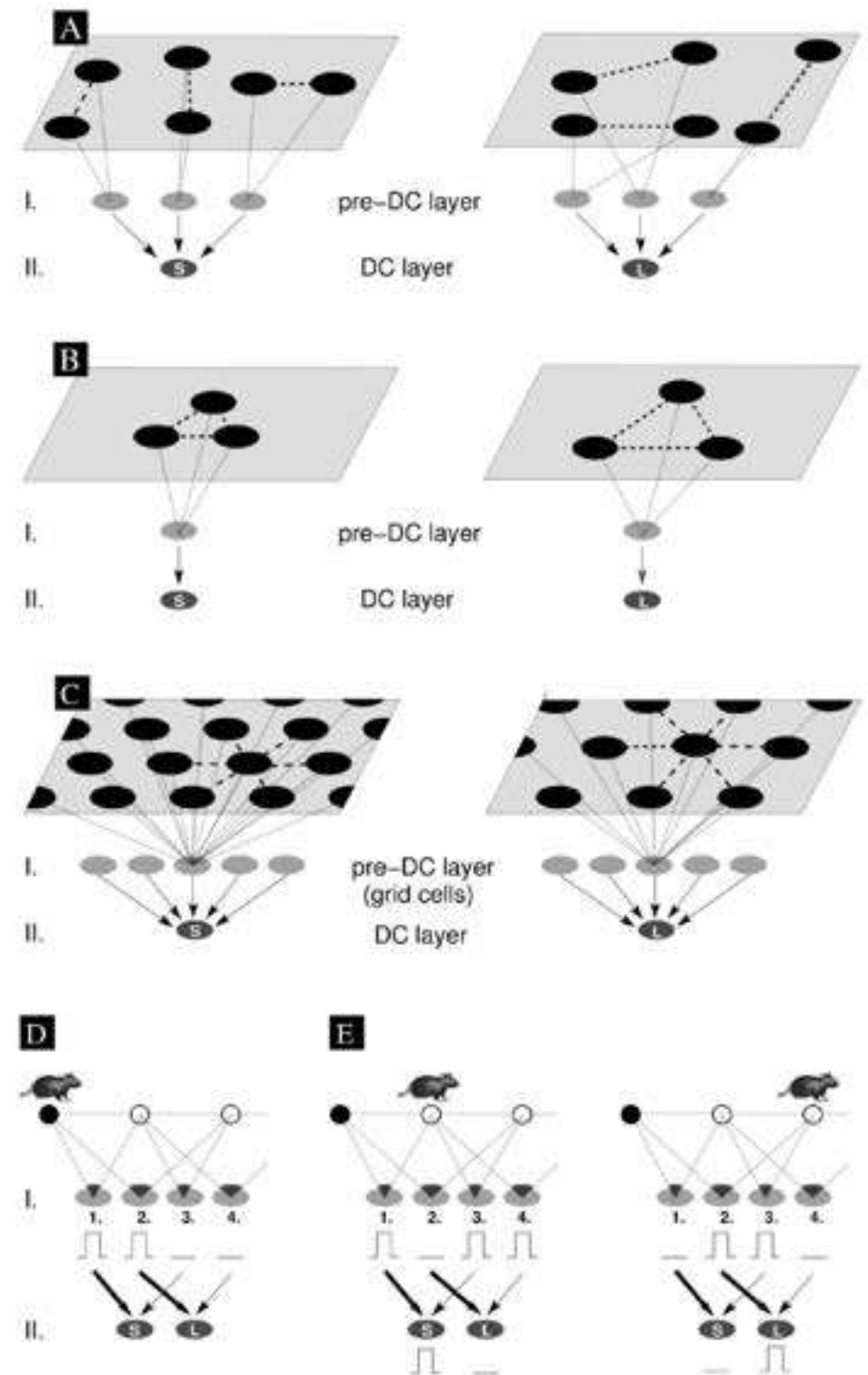


**Fig. 9.** Uncalibrated dead-reckoning error (*curve a*) versus calibrated robot positioning using sEC cell activity (*curve b*)

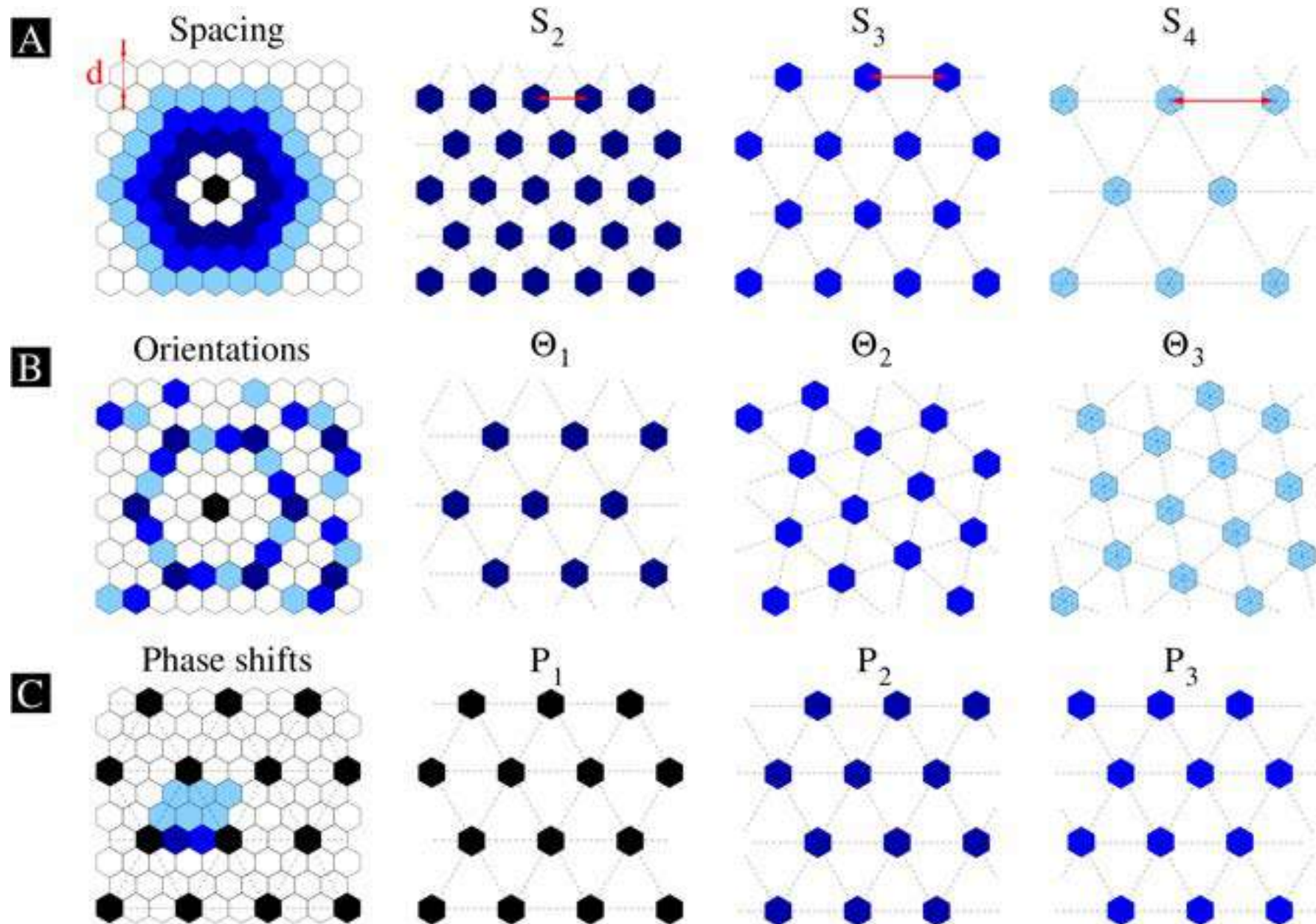


**Fig. 11.** **a** A two-dimensional view of the environment with a feeder location (*dark grey square*), and two obstacles (*white rectangles*), and an example of robot trajectory induced by the action cell activity after learning. **b** Vector field representation of the learned navigational map

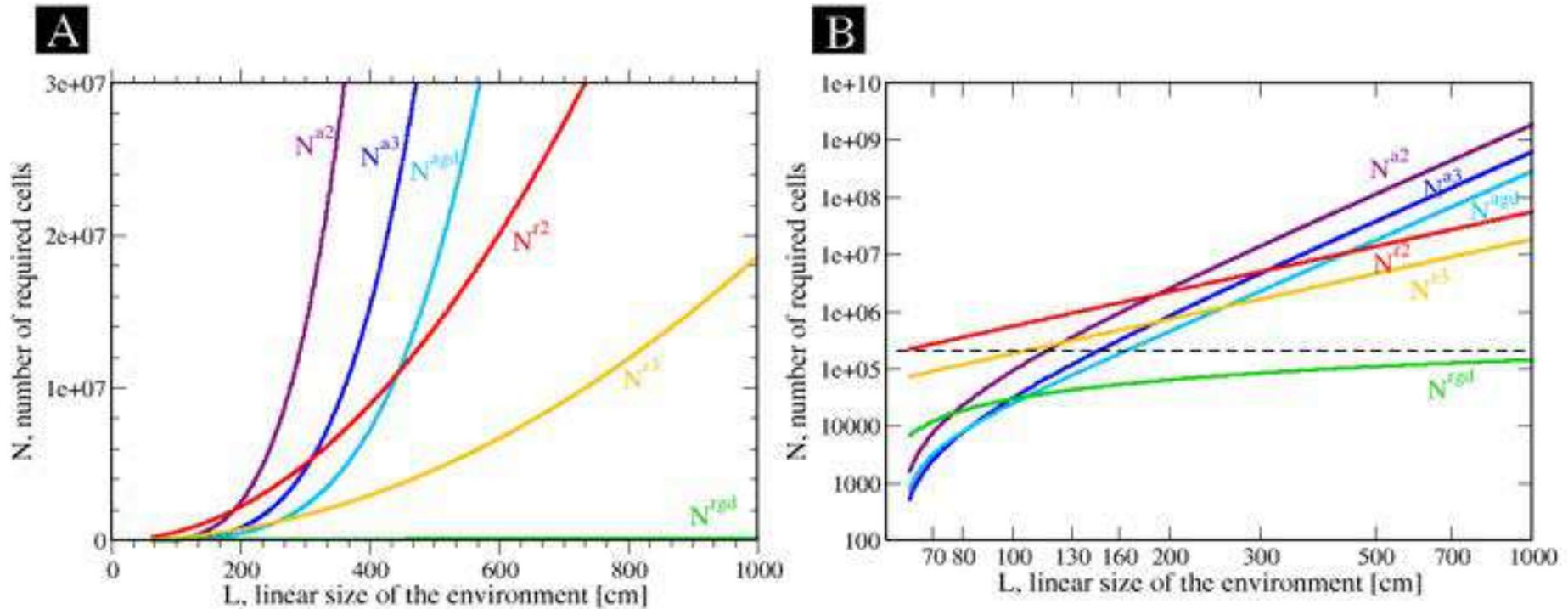
# Extraction of distance from grid cell activity



# Analytic calculations based on grid decomposition



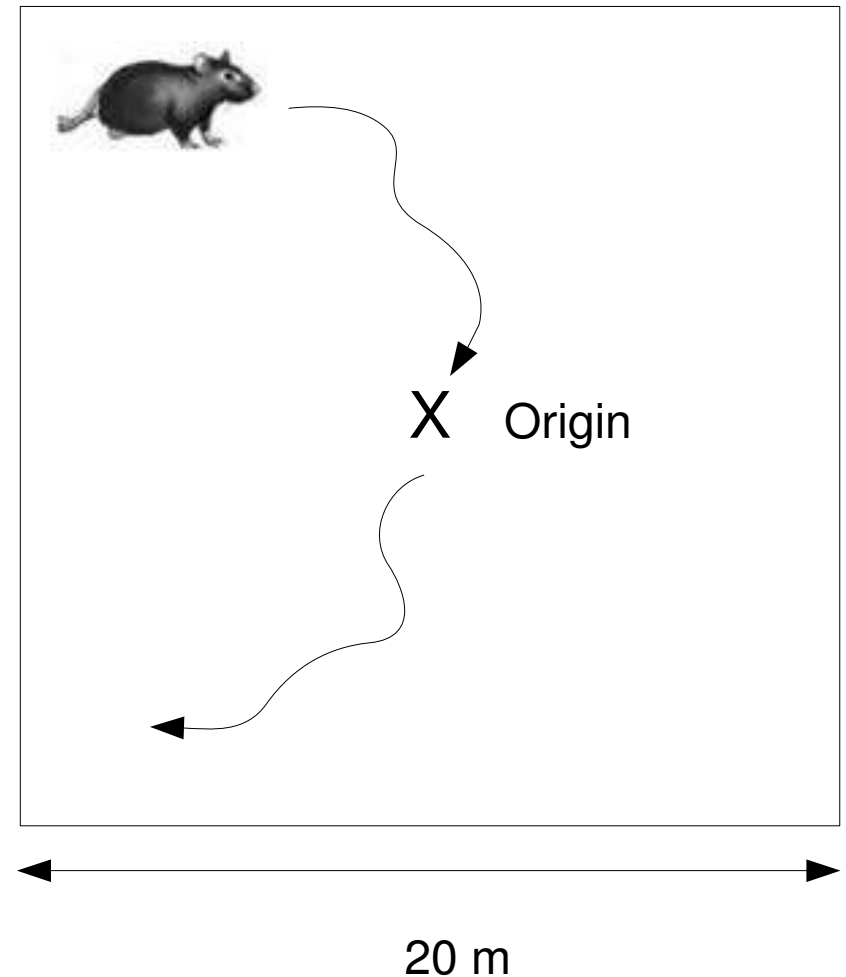
# Number of necessary cells to represent distances up to a given precision



The number of necessary cells increases as power law with the size of the environment in case of simpler solutions, only grid cells provide scalable solution, which increases logarithmically.

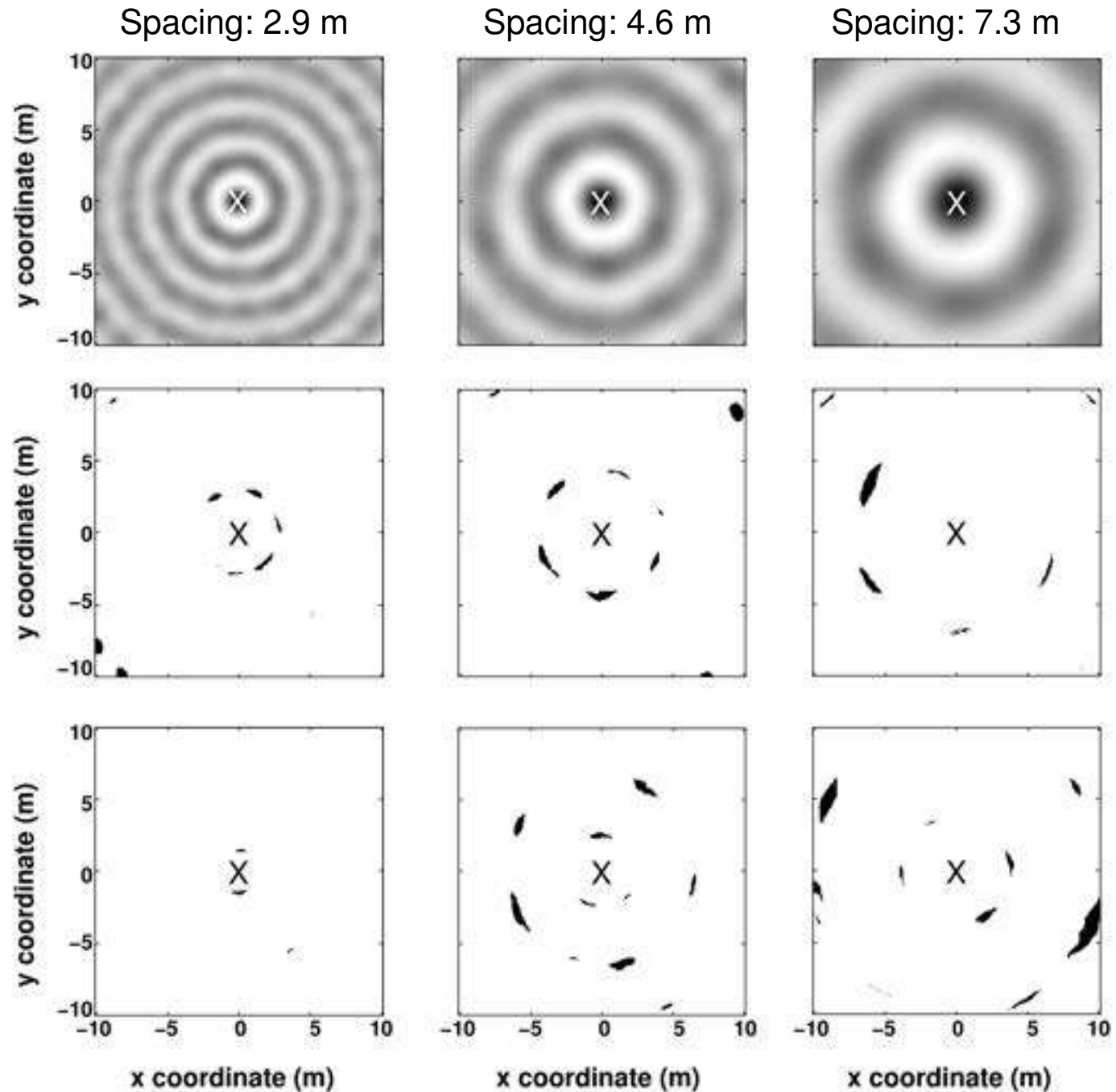
## FRAMEWORK

- The rat is exploring a 20 x 20 m arena
- When it reaches the significant place (origin), grid cell synapses get potentiated
- One-shot learning: synaptic weight becomes proportional to the firing frequency of the (presynaptic) grid cell at the origin
- The animal's distance from this point should be measured



## RESULTS

Input from grid cells





# Why it is important?

The main drive of the mammalian evolution was the conquer of the night

The path integration based navigational system of the rat works well in the darkness

Navigation in the darkness requires awareness of non-sensible objects

This could be a main step towards higher level of abstraction

# Thank you!

